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# Electrical
# .LTERNATING CURRENTS OF ELECTRICITY:

THEIR

## eneration, Measurement, Distribution, and Application.

*AUTHORIZED AMERICAN EDITION.*

BY

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WITH AN INTRODUCTION

BY

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# INTRODUCTION.

THE writings of Mr. Kapp have always interested American engineers. In fact, whenever a new branch of our science develops into attractive importance and requires "treatment," we expect that Mr. Kapp will place the matter before us in simple terms and illustrate the points of interest by examples possessing practical importance. The following pages accord with the previous writings of Mr. Kapp in this respect.

Starting with the assumption that the reader is acquainted with the behavior of steady currents in circuits whose constants are readily obtainable, he proceeds to explain the growth of a periodic current, and in so doing beguiles the student into mastering a few simple mathematical methods, a knowledge of which is fundamentally necessary.

As the volume advances its scope becomes apparent, for it treats in a simple and yet effective manner of periodic currents in general, of the phase relations of impressed and induced E. M. F's possible in simple circuits, of Alternators (somewhat in detail), of the requirements of Central Stations (briefly), of Alternating Current Motors, and finally of Multiphase Currents. One does not expect exhaustive treatment of any one of these subjects within the compass of a small book.

From an educational standpoint Chapters I. and II. are of especial importance. These forty pages might well be increased fourfold, for although they contain a summary of the elemental knowledge necessary for a correct understanding of the principles operating alternating currents in simple circuits, possessing resistance and self-induction, they do not treat of capacity effects and ʻhe problems incident thereto. In Chapter IV. we ɘ some of the formulæ for calculating induced

E. M. F.s (whether occurring in alternators or in transformers) with constants for particular cases of armature coils. Chapter V. is devoted to Machine Construction, and points out the dependence of efficiency on core wastes, giving a curve of the relation of watts per ton of core metal lost by hysteresis, to the induction density. Chapter VI. is devoted to transformers and briefly points out the necessity for careful design in this, the simplest of alternate current apparatus. The chapters on Central Stations are of interest, as they not only describe a few English plants, but also give us the author's criticisms on various distribution methods. The chapters on Alternate Current Motors and Multiphase Currents complete the little volume.

There are one or two opinions expressed by the author to which exceptions may be taken, notably the statement on page 143 that the resultant field produced by quarter-phase currents varies 40%, and that the Dobrowolski-Tesla arrangement of three phased currents produces a shifting field of more constant value than the quarter-phased or two-phased arrangement. For a discussion of these points the reader may consult *The Electrical World*, vol. xix., p. 249, Kelly; vol. xx., p. 4, Dolivo-Dobrowolski; p. 36, Kelly, Steinmetz; p. 114, C. E. L. Brown.

In general, however, Mr. Kapp's statements are clear, true, and convincing, and are of interest and value to every American engineer.

WM. STANLEY, JR.

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# Alternating Currents of Electricity:

THEIR

GENERATION, MEASUREMENT, DISTRIBUTION, AND APPLICATION.

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## CHAPTER I.

### INTRODUCTORY.

WHEN we think or speak of electric currents we are accustomed to regard them in the light of material currents, of something which flows along a path formed by the conductor, and has, therefore, a direction. We say that electricity flows along the conductor or through the conductor from the place of higher to that of lower potential; in the same way that water will flow from the higher to the lower level through a pipe. Such a view is, of course, purely conventional. As a matter of fact we do not know whether it is the positive electricity that flows in a given direction, or the negative electricity that flows in the opposite direction, or whether both electricities flow simultaneously in opposite directions, or whether there is any transfer of electricity through the wire at all. Indeed, according to modern views, there is merely transfer of energy, but not through the wire, the transfer taking place throughout the space surrounding the wire. To talk about an electric current flowing through a wire may therefore be an unscientific way of expressing our meaning, but it is a very convenient way, and, therefore, generally adopted. Now in adopting this conception of the flow of a certain thing called electricity along a predescribed path, we have also adopted the idea that this flow takes place in a direction which is perfectly well defined in each given case. We have no sense by which we can directly perceive an electric current or note its direction. It is true that if we get a shock we are made aware that a current has passed through us, but no number of shocks will help a man in the slightest degree to an understanding of the real nature of electric currents, nor enable him to determine their direction. We must be content to study, not the currents themselves, but their chemical, thermic, magnetic, and mechanical effects. Amongst other things we must also determine the direction of current by one or other of these effects. For instance, we know that a wire stretched north-south over a compass needle, and carrying a current, will deflect the needle. If the north-seeking end is deflected to the left or westward, we know by Ampère's rule that the current flows from south to north. Conversely, if the deflection is in the opposite sense, we conclude that the current is from north to south. If the current is obtained from a battery without the intervention of any piece of moving apparatus, such as a reversing key, we notice that the needle once deflected remains in that position as long as the current flows, and we naturally conclude that the current flows continuously in the same direction, that it is, in fact, a "continuous" current. Now suppose you were to notice that the needle, after remaining deflected to the left for a certain time, were to swing over to the right and to remain deflected in that position an equal time, then again swing to the left, and so take alternately these two opposite positions, you would immediately conclude tha someone had put a reversing key into your circuit and was amusing himself by working it at regular intervals. The behaviour of the needle would, in fact, have shown you that you have no longer to do with a continuous current, but that your current has become an alternating current, that is, a current which changes its direction periodically. You will notice that I have assumed that the needle has time to follow each impulse of the current, in other words, that the periodic time of the current is large in comparison with the time of oscillation of the needle. Suppose, however, that I were to work the reversing key so fast that the needle cannot follow the different impulses; in this case it will, of course, remain in its north-south position, and will have become useless as an instrument for the detection of an alternating current. We require an apparatus which will respond far more readily than a sluggish compass needle to the different current impulses which follow each other with great rapidity. To get such an apparatus, let us take an iron diaphragm, and hold near the centre of it a coil of insulated wire forming part of the circuit, or better still, an electromagnet with a laminated core; why the core should be laminated I shall explain later on. For the present it interests us to note that the poles must be in such a position that at least one of them may act on the diaphragm. Thus a ring-shaped magnet which has no free poles would not serve our purpose; a straight bar magnet, however, will do well. Now observe what happens if an alternating current is sent through the coil of this magnet. At the moment of pressing down the key to complete the circuit the battery begins to send a continuous current through the coil and the core begins to get magnetized. The magnetization grows from zero to a maximum, and retains that value until the key is lifted again, when it falls to zero. Now reverse the current and go through the same process. It is obvious that at each reversal of the current the magnetization must pass through zero, and the end of the core which is presented to the diaphragm will alternately become a north and south pole. The diaphragm will, therefore, be alternately attracted and released, or, in other words, it will vibrate, and if the period of vibration is quick enough, that is, if I manipulate the reversing key very rapidly, a musical note may be produced. Conversely, if I approach an electromagnet to a diaphragm and find that the latter is not permanently attracted, but is set in vibration and emits a musical note, then I conclude that the current which flows through the coil of the electromagnet is an alternating current, and the rapidity of the alternations, or, as it is called, the "frequency" of the current, can be judged from the pitch of the note. In explaining this experiment I have, for the sake of simplicity, assumed that the current is furnished by a battery, and that its alternating character is produced by means of a reversing key. This mechanism is, however, not an essential part of the experiment or of its explanation. The essential part is that the current shall grow from zero to a maximum, and diminish again to zero, then change its direction and grow to a negative maximum, diminish to zero, then become positive again, and so on. Such a current is produced by a certain class of electric machines called "alternators," which will occupy us a good deal during this lecture. But before entering into this subject I wish to show you experimentally the fact that an alternating current can produce these oscillating or wave-like magnetic effects which I described a moment ago. The apparatus I shall use in my illustrations is extremely simple. I have here a small electromagnet of the kind used in connecting arc lamps to alternating current circuits, and which is technically termed a "choking coil." For a diaphragm I use the bottom of an ordinary biscuit tin, and you will observe that when I approach one end of the choking coil to the biscuit tin there is emitted a sound which can be heard all over the room. The sound is not exactly a clear musical note, because, as might have been expected in a rough-and-ready apparatus of this kind, the elasticity of the diaphragm is by no means perfect. But such as the sound is, it serves quite well to show that the diaphragm is set vibrating by the current, and, in fact, every telephone receiver exemplifies the same action.

The study of alternating currents is greatly facilitated by a rational and simple manner of representing them graphically. There are various ways in which we can so represent not only alternating currents, but any quantity which varies periodically. The most obvious way of representing an alternating current is by drawing a curve, the two co-ordinates of which represent time and the instantaneous current strength. In Fig. 1 the time

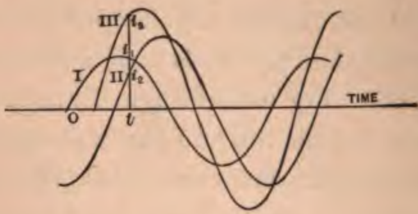


FIG. 1.

is measured on the horizontal, and the current strength on the vertical. We thus obtain a wavy line which cuts at regular intervals through the axis of abscissæ. These are the points of reversal when the current strength is maximum, positive where the line lies above, and negative where it lies below the axis. The exact shape of the curve depends on the construction of the machine which produces the alternating current; but I may at once say that in nearly all the theoretical investigations of alternating currents it is assumed that the curve follows, or rather represents, a sine function, and that this assumption is sufficiently near the truth for all practical purposes. All of you know, of course, what a sinusoidal curve is, and I need, therefore, not explain it at length. As, however, the way of plotting a sinusoidal curve brings me to a second method of representing an alternating current graphically, I must say a few words about it. Imagine yourself standing some distance in front of a steam engine in a line with the axis of the cylinder, and looking at the crank pin. The latter will then appear to be moving up and down, making equal excursions to both sides of the centre of the crank shaft. You will, in fact, see the projection of the crank on a vertical, and the length of this projection at any instant is equal to the length of the crank multiplied with the sine of the angle which the crank makes at that instant with the horizontal. The angle is, of course, the product of the angular velocity and the time; and since the angular velocity is constant, you will also obtain a sine curve by plotting the time on the horizontal and the projection of the crank on the vertical. The curve I in Fig. 1 has been so obtained. We may, however, save ourselves the trouble of plotting this curve, for we can represent the alternating current more directly by the projection on the vertical of a line OI (Fig. 2) revolving with a constant angular speed round the fixed centre O.

The length of OI represents to any convenient scale the maximum value of the current, or the crest of the current wave, and its projection represents its instantaneous value. You see that for


FIG. 2.

half a revolution this value is positive, and for the other half of the revolution it is negative.

In this diagram, which is called a "clock diagram," we must therefore make a projection in order to find the instantaneous value of the current. This is less laborious than the plotting of a sine curve, but it is possible to represent the current in a still more simple way. Those of you who are familiar with Zeuner's valve diagram will immediately see how this can be done. Instead of drawing the circle round O as centre, we draw it passing through O. The diameter of this circle (Fig. 3)

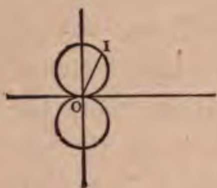


FIG. 3.

represents to any convenient scale the maximum value of the current. Then the instantaneous current is given directly by the length of the revolving line between O and the circle. To obtain the negative values of the current, we reproduce the circle on the opposite side; this in the figure is shown dotted.

To illustrate the use of any of these graphic methods of representing alternating currents, let us suppose that we have to solve the following problem:—We have an iron core wound with two independent coils, each carrying an alternating current. The two currents shall have the same frequency, that is to say, the time which elapses between two succeeding positive maxima or negative maxima shall be the same for both currents, but the maxima in the two currents shall not occur at the same moment. In other words, the phase of one current shall lag behind that of the other, just as in a two-cylinder steam engine one crank lags behind the other. Now the problem we have to solve is: what will be the magnetization of the core at any instant? To find this we must of course know the instantaneous value of the exciting power, or the ampère turns resulting from the action of both currents combined; we must, in fact, find what resultant current acting alone will have the same effect as the two given currents acting together. Let, in Fig. 1, the curves I and II represent the two currents, or better still the ampère turns of these currents, then the ampère turns of the resultant current are found by plotting the algebraical sum of the ordinates. Thus we obtain curve III. It is self-evident, and needs, therefore, no elaborate proof, that this curve can also be obtained from Fig. 2 if in that figure we draw a parallelogram of currents (precisely in the same way as in mechanics we draw a parallelogram of forces), and use the resultant O III to plot the sine curve. You see that we can combine currents in the same way as mechanical forces. I have proved this for the case that the currents flow in two independent coils, but a glance at Fig. 4 will show you that it

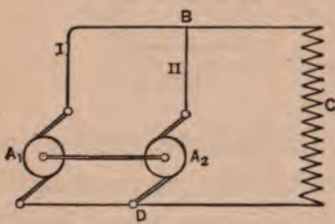


FIG. 4.

also holds good if the two currents are sent through the same coil. Here we have two machines, A1 and A2, mechanically coupled, and therefore producing currents of the same frequency. These currents, I and II, flow into one circuit containing a coil C. It is evident that in the circuit BCD there flows only one current, which is the algebraic sum of I and II.

Now let us change the arrangement to that shown in Fig. 5. Here we have to do with only a single current, for both machines and coil C are coupled in series; but we have to do with two elec-

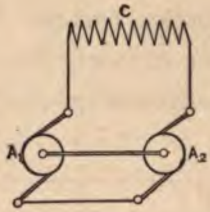


FIG. 5.

tromotive forces, namely, those of the machines. I assume that the coil C in itself has no electromotive force. In this case also it is self-evident that the current which will be forced through C is due to the algebraic sum of the two electromotive forces, and that all I have said about the determination of the resultant current is directly applicable to that of the resultant electromotive force. In other words, we may use any of the three graphic methods of representing currents also for representing electromotive forces.

These graphic methods of investigation, and es- those based on the clock diagram, are so useful and so simple that I shall employ them frequently in the course of these lectures in preference to analytical methods, and it is therefore expedient to familiarize you at the outset with the clock diagram. For this purpose I select, by way of example, a case which is very frequently met with, and which is represented by Fig. 6. Lest you should

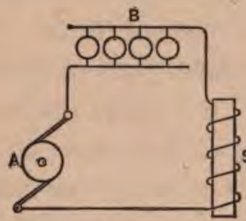


FIG. 6.

think that this case has merely theoretical importance, I may at once say that a certain deduction which flows naturally from its consideration is of great practical importance in motors driven by multiphase currents, since on it depends the starting torque of such motors. If you compare Figs. 5 and 6 you will find that they only differ in this: that an electromagnet S has been substituted for the machine A2. The circuit represented in Fig. 6

consists of a machine A, giving an alternating electromotive force, a resistance B, consisting of a bank of glow lamps, and an electromagnet S. This electromagnet has a property which is technically called "self-induction," and, before going further, I must briefly explain to you what is meant by self-induction. You know that an electromotive force is set up in a wire whenever the wire cuts across magnetic lines of force. Since the wire must necessarily form part of a closed circuit (for if the circuit were not closed there could be no current), the cutting of lines must be accompanied by an increase or decrease in the number of lines or total induction threading through the circuit, and we may therefore also say that whenever the total induction through a circuit changes, there is an electromotive force set up in the circuit which is the greater the more rapid the change. In fact, the rate of change, that is, number of lines added or withdrawn per second, multiplied with the num- of turns of wire, gives the electromotive force p in the coil. Going back to Fig. 1, we have at the curve I represents the current as a f the time. Suppose there is no other coil wound over the core, then the ordinates of the curve represent to a suitable scale also the exciting power on the core, and it is obvious that the magnetization of the core, or, to speak correctly, the total induction passing through it, will change more or less in accordance with the curve I. If the permeability were constant, the induction would be strictly proportional to the exciting power, and by the selection of a suitable scale the curve representing induction could be made to coincide with the current curve I. Now for low values of the induction, say between zero and 3,000 or 4,000 lines per square centimètre, we may regard the permeability of soft, well-annealed wrought-iron as approximately constant, and if we do not press the induction beyond this point, we may without any great error assume that the current curve I also represents the total induction through the core. For the points where the current passes through zero, and which momentarily interest us the most, the assumption is, of course, quite correct. But if the curve I represents the total induction, then the geometrical tangent to it at any point represents the change of induction in unit time, or, as I said just now, the rate of change of induction at the particular moment represented by the point on the curve. Thus, reading off the time on the horizontal axis, we can, by drawing the tangent to the current curve at the corresponding points, find the rate at which the total induction changes at each moment. I said just now that the rate of change, multiplied with the number of turns in the coil, gives the electromotive force generated at any instant in the coil, and it will now be clear to you that this electromotive force, which we call the "electromotive force of self-induction," must be proportional to the geometrical tangent to the current curve. The steeper this line, the greater is the electromotive force. Thus you see that when the current is either a positive or negative maximum, the tangent is horizontal, and therefore at those moments the electromotive force of self-induction is zero. On either side of maximum current it has a definite value, but this value is positive on one side and negative on the other side of maximum current, since the slope of the tangent 'anges from upward to downward when passing "oint. Where the current curve intersects the horizontal axis, the slope of the tangent is evidently greatest, and we therefore see that the electromotive force of self-induction is a maximum when the current passes through zero, and it is itself zero when the current is a maximum. This then is, in general terms, the relation between the current curve and the curve giving the electromotive force of self-induction. It remains yet to determine the exact nature of the latter. We have seen that the ordinates of the electromotive force curve are proportional to the geometric tangent drawn to the current curve. Now how do we draw the tangent to the point A for instance? We draw a straight line through this point, and one very near it, on the current curve. To speak correctly, I should say infinitely near it. At this infinitely near point the current will have increased from $i$ to $i + di$, and the time from $t$ to $t + dt$. The ratio of $di$ to $dt$ is therefore equal to the geometrical tangent at A. But this ratio is the differential quotient of the current in respect to time, and we thus find that the curve giving the electromotive force of self-induction is the first differential of the current curve.

I have up to the present entirely avoided the use of mathematics, but now it becomes necessary to introduce a few simple formulæ. Going back to Fig. 2, suppose that the radius OI, the projection of which gives the instantaneous value of the current, makes $n$ complete revolutions per second. Its angular speed is then $\omega = 2\pi n$, and its angular position at the time $t$ is $\alpha = \omega t$, counting the time from the moment that the radius is horizontal. Let I be the length of the radius, which also represents the maximum of the current strength, or crest of the wave, then the instantaneous value of the current at the time $t$ is

$$i = I \sin \omega t \ldots\ldots\ldots\ldots\ldots\ldots (1),$$

and the electromotive force of self-induction at that moment is

$$e_s = L\omega I \cos \omega t \ldots\ldots\ldots\ldots\ldots\ldots (2),$$

L being a coefficient which depends on the permeability of the core, the magnetic reluctance of the whole magnetic circuit, and the number of turns in the coil. At the time when the current passes through zero we get the maximum value of the electromotive force, which is

$$E_s = L\omega I \ldots\ldots\ldots\ldots\ldots\ldots (3),$$

and we can also write the equation for the instantaneous value of the electromotive force of self-induction in the form

$$e_s = E_s \cos \omega t,$$

or

$$e_s = -E_s \sin\left(\omega t - \frac{\pi}{2}\right) \ldots\ldots\ldots\ldots (4),$$

from which you will see that this value may also be graphically represented by a sine curve, but lagging behind the current curve by 90 degrees. To obtain the electromotive force curve, which is shown dotted in Fig. 7, we must therefore imagine

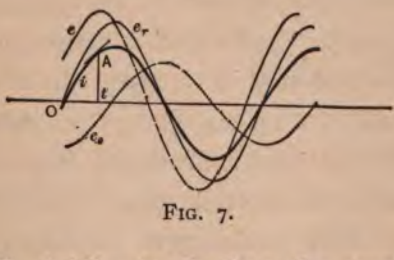


FIG. 7.

a crank $OE_s$ rigidly attached to the crank OI (Fig. 8) at an angle of 90 degrees, and we must plot the projections of this second crank on the vertical as ordinates in Fig. 7. A study of this diagram (Fig. 7) will help you materially to an understanding of the phenomena of self-induction. As time progresses, from left to right you see that at first the current is positive and increases. But self-induction opposes the increase, and its electromotive force is therefore negative. The dotted curve is below the axis. This opposition becomes fainter as the current approaches its maximum value, since the rate of change of the induction becomes less and less. When the current has reached its positive maximum, the rate of change has become zero, and the opposition of self-induction has vanished. The dotted curve passes through the axis. A moment later, the current is still positive, but is now decreasing. Again self-induction opposes the change; its tendency is to keep the current up at its maximum strength. The electromotive force of self-induction tries to push on the current; it is positive, and the dotted curve rises above the horizontal. This tendency to push on the current increases until the current has become zero, and begins to flow in the reverse direction. The negative current is now opposed by the positive electromotive force of self-induction, but the opposition grows fainter as the current grows stronger, and so the see-saw of pushing on and checking back the current is kept up.

The point which interests us most is what must be the electromotive force given by the machine A, in Fig. 6, to produce the current shown by the curve in Fig. 7. To simplify our investigation we shall assume that the ohmic resistance of the coil S and machine A is negligible in comparison with the ohmic resistance of the bank of lamps B, or that it is included therein; also that the only part of the circuit having self-induction is the coil S. Then it is immediately obvious that a voltmeter placed across the terminals of this coil will indicate the electromotive force of self-induction, and a voltmeter placed across the lamps will indicate the electromotive force corresponding to the product of current and the resistance of the bank of lamps. But it is not immediately obvious that the sum of these two readings will give us the electromotive force as measured by a voltmeter across the terminals of the machine, and I will show you presently, by theory and by experiment, that this is not the case. Assuming the resistance of the bank of lamps to be a fixed quantity $r$, it is clear that the instantaneous lamp volts equal the product $r \times i$, and that they can be represented by a sine curve $e_r$ of the same phase as the current curve. In Fig. 8


FIG. 8.

the radius of maximum lamp volts $OE_r$ must therefore coincide with the radius of maximum current OI. The radius of maximum volts of self-induction is $OE_s$, and this, as I have already shown, lags behind the current radius by 90 degrees. To find the machine volts at any instant, we must combine the curves $e_r$ and $e_s$, but remember to take the latter with the opposite sign, for the self-induction opposes the current. This gives us the curve $e$ in Fig. 7. To find the machine volts from Fig. 8 we have to draw a radius of such length and position that it may be regarded as the resultant of the lamp volts $E_r$, and an electromotive force diametrically opposed to that of self-induction. We prolong, therefore, the line $E_sO$ beyond O, and make $OE_s' = OE_s$. Completing the parallelogram, we thus find the resultant OE, which gives us the maximum machine volts, or "impressed electromotive force."

The diagram (Fig. 8) is very instructive. In the first place, it enables us at once to find an expression for the angle of lag $\varphi$. You see that the tangent of this angle is given by the ratio of the electromotive force of self-induction to that usefully expended over the lamps. I must here remark that when I speak of electromotive force and current I mean, for the present, always their maximum values. Retaining the notation previously employed, we have, therefore—

$$\tan \varphi = \frac{L\omega I}{rI},$$

$$\tan \varphi = \frac{L\omega}{r} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (5).$$

Next we can find an expression for the current as a function of the impressed electromotive force and the constants of the circuit. Since the triangle $OE_rE$ is rectangular, we have

$$E^2 = E_r^2 + E_s^2,$$

or, with our previous notation—

$$E^2 = r^2 I^2 + L^2 \omega^2 I^2,$$

$$I = \frac{E}{\sqrt{r^2 + L^2 \omega^2}},$$

$$I = \frac{E}{r} \frac{1}{\sqrt{1 + \left(\frac{L\omega}{r}\right)^2}} \ldots\ldots\ldots\ldots (5a).$$

If we had to do with a continuous current, its equation would be $I = \frac{E}{r}$. Since the term under the square root must, under all circumstances, be larger than unity, the current produced by an alternating electromotive force must always be smaller than the current which an equal but continuous electromotive force would produce in the same circuit.

The term $\sqrt{r^2 + L^2 \omega^2}$ is called the "impedance" of the circuit, and $L\omega$ its "inductance." As an aid to memory, I reproduce in Fig. 9 Dr. Fleming's diagram, in which these terms are recorded. You have seen that Ohm's law is not applicable to alternate current circuits, but if we substitute the impedance for the ohmic resistance, this law becomes applicable.

I have yet to explain the meaning of the quantity which we called L, and which we introduced in order to take account of the number of turns in the coil, and other properties of the circuit. Of course, most of you will long ago have recognized in this **L** the usual coefficient of self-induction, but, for

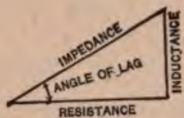


FIG. 9.

the sake of completeness, I must prove this. There are various definitions for the coefficient of self-induction, but the following will serve my purpose:—"The coefficient of self-induction is the ratio between the counter electromotive force in any circuit and the time rate of variation of the current producing it." * In symbols—

$$e_s = \mathrm{L}\frac{di}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots(6),$$

* Sumpner, *The Variation of Coefficients of Induction. Phil. Mag.*, June, 1887, p. 453.

and substituting $i$ from equation (1), we have—

$$e_s = \mathrm{L}\omega\mathrm{I}\cos\omega t \quad \ldots\ldots\ldots\ldots\ldots\ldots(2),$$

which is identical with equation (2), and shows that the L we then introduced is indeed the coefficient of self-induction.

# CHAPTER II.

## MEASUREMENT OF PRESSURE, CURRENT, AND POWER.

When showing you the last experiment, I had occasion to use a voltmeter, and the question we have now to consider is what is the relation between the reading of the instrument and the maximum electromotive force in the circuit. That the reading must be less than the true maximum is obvious, but less by how much?

To answer this question we may use the analytical or the geometric method. I give the former in Appendix I. of this chapter and the latter, which is due to Mr. Blakesley, in Fig. 10. A Cardew voltmeter measures not directly volts, but simply the amount of heat developed in its wire per unit of time. The rate at which heat is developed at any instant is the product of the instantaneous current and the instantaneous volts; but as the current passing through the wire is proportional to the volts, the rate at which heat is developed is proportional to the square of the instantaneous volts, that is, to $O_e$ squared in Fig. 10, if by OE we represent the maximum volts. Now, to find the general effect of a large number of succeeding instantaneous voltages on the voltmeter, we have to draw the projections $O_e$ of OE for a large number of positions. Let us take these positions in pairs, such as OE and OE′, with an angular interval of 90 degrees between


FIG. 10.

them. It is evident that for such pair the sum of the squares of the projections is equal to the square of the maximum voltage, and that the mean voltage is $\frac{1}{2}$ $E^2$. This is independent of the actual position of each pair, and is, therefore, the mean value of all the possible pairs. The volts read on the Cardew (or any other instrument, the action of which depends on the square of the voltage) must therefore be multiplied by the square root of 2 in order to get the maximum volts; or in symbols, if by $e$ we represent the volts shown by the instrument, and by E the maximum volts—

$$e = \frac{E}{\sqrt{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7).$$

At the Paris Congress, in 1889, it has been decided to call $e$ the "effective" volts.

The same reasoning which I have here applied to the measurement of pressure can also be applied to the measurement of current, provided that the measuring instrument is of a kind in which the movable part is subjected to a force varying as the square of the current. Thus a Siemens' dynamometer, a Thomson ampère-balance, and similar instruments, will indicate the "effective current"—

$$i = \frac{I}{\sqrt{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(8).$$

It is important to osberve that this is not the same thing as the mean or average current. To understand the distinction, let us first settle what quantity we call the mean current. Imagine an alternating current commutated each time it passes through zero, and let the unidirected but pulsating current thus obtained pass through an electrolytic apparatus, say, for instance, a copper voltmeter. The weight of copper thrown down in unit time is a measure of the "mean strength" of our pulsating current. The mean current strength thus defined is about 90 per cent. of the effective current strength, so that to get the true mean current we must multiply the reading of the Siemens' dynamometer by 90. The proof for this is given in Appendix II. of this chapter.

We have seen that the measurement of pressure and current is a very simple matter; but the measurement of power, to which I must next draw your attention, is not quite so simple as with continuous currents. You know that if we wish to determine the power given to a circuit by a continuous current we have merely to observe the ampères and the volts, and multiply them to get the watts. If we divide the watts by 746, we get the result in horse-power. With alternating currents this is not quite so simple, and if we were to compute the power in this manner, our results would be generally too large, and never too small. The reason, of course, is that the instant of maximum ampères does generally not coincide with the instant of maximum volts. To get the true power we must integrate the product of the instantaneous volts and ampères over a complete cycle, and divide by the time required to perform the complete cycle. The matter is treated analytically in Appendix III. of this chapter, while here I treat it graphically,


FIG. 11.

using again Mr. Blakesley's method. In Fig. 11 OE and OI represent electromotive force and current at any given moment, OE′ and OI′ their position a quarter period later. The current lags behind the electromotive force by the angle $\varphi$.

Using small letters for the projections of E and I on the vertical, the mean value of the power for the two positions is obviously $ei + e'i' \div 2$.

From the diagram, the following equations are obvious:—

$$ei = \text{EI} \sin \alpha \sin \beta.$$
$$e'i' = \text{EI} \cos \alpha \cos \beta.$$

Combining these we find the mean power—

$$w = \frac{\text{EI}}{2} (\cos \alpha \cos \beta + \sin \alpha \sin \beta).$$

$$w = \frac{\text{EI}}{2} \cos (\alpha - \beta).$$

$$w = \frac{\text{EI}}{2} \cos \varphi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9),$$

and this is the same for every pair the position of which differs by 90 degrees. It is, therefore, the true equivalent power.

Since $e = \frac{\text{E}}{\sqrt{2}}$ and $i = \frac{\text{I}}{\sqrt{2}}$, we have also—

$$w = ei \cos \varphi \quad \ldots\ldots\ldots\ldots\ldots (10),$$

where $e$ and $i$ are the volt and ampère readings, as obtained by our usual instruments.

*You* see we have first to determine the "apparent" watts as if we had to do with a continuous current, and then to get the true watts we must multiply by the cosine of the angle of lag. It is, however, not always easy to determine the angle of lag, and to avoid the labour and possible errors of such a determination, various instruments have been invented for the direct measurement of power, which are called wattmeters. The best-known form of wattmeter is constructed similarly to a Siemens' dynamometer. The fixed coil, containing a few turns of thick wire, is connected in series with the main circuit, and the movable or suspended coil, containing many turns of fine wire, is connected as a shunt to the main circuit. A non-inductive resistance is put in circuit with the movable coil to reduce the self-induction of the shunt circuit. (For theory of wattmeter, and corrections to be applied, see Appendix IV., following.)

# APPENDIX I.

Voltmeter absorbs in time T the energy—

$$\int_0^T \frac{e^2}{r} dt.$$

$$wT = \frac{1}{\omega} \frac{E^2}{r} \int_0^T \sin^2 (\omega t)\, d\, (\omega t).$$

$$wT = \frac{1}{\omega} \frac{E^2}{r} \left[ \frac{\omega t}{2} \right]_0^T.$$

$$wT = \frac{T}{2} \frac{E^2}{r}.$$

And since

$$w = \frac{e^2}{r},$$

$$e = \frac{E}{\sqrt{2}}.$$

# APPENDIX II.

Mean current, as determined by electrolysis, is coulombs divided by time—

$$\frac{T}{2}\,\omega = \pi.$$

Mean current—

$$c = \frac{1}{\frac{T}{2}}\int_0^{\frac{T}{2}} i\,dt.$$

$$c = \frac{2}{T}\int_0^{\frac{T}{2}} I \sin(\omega t)\,dt.$$

$$c = \frac{2I}{\omega T}\int_0^{\frac{T}{2}} \sin(\omega t)\,d(\omega t).$$

$$c = \frac{2I}{\omega T}\Big[\cos \omega t\Big]_{\frac{T}{2}}^{0}.$$

$$c = \frac{I}{\pi} \times 2.$$

$$c = \frac{I}{\frac{\pi}{2}}.$$

The effective current is—

$$i = \frac{I}{\sqrt{2}};$$

therefore

$$c = i\frac{\sqrt{2}}{\frac{\pi}{2}},$$

or very nearly

$$c = 0{\cdot}9\,i.$$

---

## APPENDIX III.

Work done by current during one cycle is $w$T, and per second it is—

$$w = \frac{1}{T}\int_0^T eidt.$$

$$w = \frac{IE}{\omega T}\int_0^{2\pi} \sin a \sin (a + \varphi)\, da.$$

$$w = \frac{IE}{2\pi}\int_0^{2\pi} \cos \varphi \sin^2 ada + \sin \varphi \sin a \cos ada.$$

$$w = \frac{IE}{2\pi}\left[\cos \varphi \left(\frac{a}{2} - \frac{1}{2}\sin a \cos a\right) + \sin \varphi \left(\frac{1}{2}\sin^2 a\right)\right]_0^{2\pi}.$$

$$w = \frac{IE}{2\pi}\Big[\cos\ \varphi\pi\Big] \text{ or } w = \frac{IE}{2}\cos\varphi \text{ or } w = ie \cos \varphi.$$

## APPENDIX IV.

Let $\varphi$ be the angle of lag in the circuit, the power given to which is to be measured, and let $\delta$ be the angle of lag in the fine wire coil of the wattmeter, due to its self-induction. Let I be the current through the thick wire coil, and $i$ the current through the fine wire coil, then the power indicated by the wattmeter, if the currents were steady, would be $KriI$ where K is the coefficient of the instrument, and $r$ the resistance of the fine wire coil. The true watts of the alternating current of E volts are—

$$W = IE \cos \varphi.$$

The indicated watts are—

$$W' = K \text{ (Reading)}.$$
$$W' = Kri \cos(\varphi - \delta)\, I = KE \cos \delta \cos(\varphi - \delta)\, I.$$

Therefore, to get true watts, we must multiply the watts indicated by

$$\frac{\cos \varphi}{\cos \delta \cos(\varphi - \delta)},$$

which expression can also be written in the form—

$$\frac{1 + \tan^2\delta}{1 + \tan\delta \tan\varphi}.$$

If the wattmeter has no self-induction, $\delta = 0$, and no correction is required. Again, if the self-induction of the wattmeter is equal to that of the circuit to be measured—

$$\tan\delta = \tan\varphi, \text{ and } \frac{1 + \tan^2\delta}{1 + \tan\delta \tan\varphi} = 1.$$

In this case also no correction is required. In all other cases the corrections given in this formula must be applied.

# CHAPTER III.

## CONDITION OF MAXIMUM POWER.

It is important to investigate the conditions under which we can obtain a maximum of power in a given circuit. This is the deduction of which I have spoken a little while ago as flowing naturally from the investigation of the case represented by Fig. 6. Here we have an alternating current machine, a self-induction, and a bank of lamps. The self-induction we cannot diminish, and the machine volts we cannot increase. How must we manage our lamps to get a maximum of power into them, and therefore to get a maximum of light out of them? Without entering into any lengthy mathematical investigation, you can see at once that the ohmic resistance of the bank of lamps will have a great deal to do with the amount of power usefully expended. If the resistance is very high, the lamps will get very nearly the whole of the machine volts, but then the current will be small. If, on the other hand, we lower the resistance too much in our desire to get a large current, we shall have to sacrifice nearly the whole of the pressure, since the self-induction, which now is fed by a large current, will choke back most of the available voltage. You see that either too little or too much resistance is bad, and we have to find that resistance which will give us the best effect. This will be the case when the volts over the lamps equal the volts over the self-induction, either being about 70 per cent. of the machine volts. I give the analytical proof in an appendix to this chapter and the geometric proof by means of the clock diagram (Fig. 12). Let, in this figure, the circle represent the given machine volts, and let $OE_s$ be the volts of self-induction corresponding to the current OI. Then the tangent of the angle at I is obviously equal to $L\omega$ (by equation (3)). The power given to the lamps is (by equation (9)) $w = \frac{1}{2}$ IE cos $\varphi$, that is, the projection of the volt radius on the current radius, multiplied by the current, and divided by 2. But the projection of the machine volts OE gives the lamp volts $OE_r$, and the current is proportional to the volts of self-induction (see triangle $OIE_s$), so that we can also say the power is proportional to the product of lamp volts and self-induction volts (that is, to the shaded area in Fig. 12),

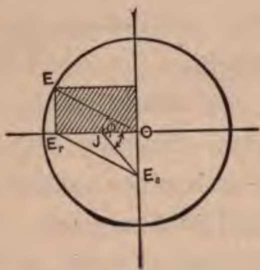


FIG. 12.

and our problem can also be stated in these terms:— Find that position of OE for which the shaded area becomes a maximum. Obviously this will be the case when the line OE forms an angle of 45 degrees with the horizontal; in other words, when the current lags by one-eighth of a period behind the impressed electromotive force, and when the volts measured over the self-induction equal the lamp volts. This condition will be fulfilled when the resistance of the bank of lamps is

$$r = L\omega.$$

In order to simplify the explanation, I have assumed that in Fig. 6 the resistance, the self-induction, and the seat or source of electromotive force, are different and distinct parts of the circuit. This was, however, not essential. We could, for instance, assume the self-induction to be part and parcel of the machine, or even of the bank of lamps, and yet our result would have been the same. Nay, more. Suppose we take away the lamps and put in their place a series wound dynamo with well-laminated field magnets. Whichever way the current is sent through the machine it will always revolve in the same direction, and the alternating current must, therefore, set it in motion. Now, imagine the period of the current very long, in fact, so long that the electromotive force of self-induction of the magnet coils and armature may be neglected. Then the only force opposing the current in its flow through the armature is the counter electromotive force developed by the latter, which, at constant speed of rotation, is proportional to the field strength. Now, if we do not excite the magnets strongly (and on account of hysteresis and other losses it is advisable to work with a low induction), we may consider the field strength to be proportional to the current, so that the only force which opposes the current will in this case be at all times proportional to the current strength. It will, in fact, be the same kind of opposition as is produced by ohmic resistance, but with this difference, that, instead of converting the electric power into heat in the lamps, we convert it into mechanical power, which may be taken off the spindle of the motor. So far the motor, although supplied with an alternating current of very long period, will work exactly as if it were joined to a continuous current circuit. But now let us increase the frequency of a current, that is to say, let us shorten the period and have more and more current waves per second. This will add to the counter electromotive force of the motor (which is useful, because accompanied by the giving out of mechanical power) another electromotive force which is entirely useless, namely, the electromotive force of self-induction. This must considerably decrease the power obtainable from the motor, first, because the current strength has been decreased by its action; and, secondly, because with the electromotive force of self-induction now having become a large quantity, a considerable lag of the current behind the machine volts must take place. A few years ago I experimented with a motor of this kind, and found that the power obtainable from the machine when coupled to an alternating current circuit was only about one-sixth its normal power on a continuous current circuit. In this motor the self-induction was far too large as compared with its counter electromotive force. By our rule, the electromotive force of self-induction should have been equal to the counter electromotive force. In this case the motor would give about 70 per cent. of the power it could develop with a continuous current. If it were possible to make motors which fulfill the condition I have explained, it would be a very easy and practical solution of the problem how to make the existing lighting stations which supply alternating current available for the distribution of power; but I doubt very much the possibility of this solution of the problem. The self-induction of such a motor must always be enormously high, but a way in which we can at least approach the best condition of working is by lowering the frequency and increasing the rotary speed of the motor. I have devoted some time to this method of working alternate current motors, because it has already acquired practical importance, not indeed in the regular working of these machines, but in the starting of them. The Ganz motor is started without a load, as if it were an ordinary continuous current machine, and after it has acquired a certain speed it suddenly begins to work as an alternating current machine. When in this condition the load can be thrown on, and the machine will even stand a certain amount of excess load.

# APPENDIX.

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The power is $w = \frac{1}{2}$ IE cos $\phi$, or substituting for cos $\phi$ the value $\frac{r}{\sqrt{r^2 + \omega^2 L^2}}$, and for I the value $\frac{E}{\sqrt{r^2 + \omega^2 L^2}}$, we have also—

$$w = \frac{1}{2} E^2 \frac{r}{r^2 + \omega^2 L^2},$$

$$w = \frac{1}{2} E^2 \frac{1}{r + \frac{\omega^2 L^2}{r}}.$$

The variable is $r$, and to find for which value of $r$ the power $w$ becomes a maximum, we resolve the equation $\frac{dw}{dr} = 0$, and find $r = \omega L$, and the maximum power—

$$w = \frac{1}{4} \frac{E^2}{r},$$

or, if by $e$ we represent the effective voltage, such as would be indicated on a Cardew voltmeter, we have also—

$$w = \frac{e^2}{2r}.$$

The analogy with the well-known rule for maximum power from a source of continuous current is remarkable. According to this rule, maximum power will be developed in the external circuit, if its resistance is equal to the resistance of the battery or machine which gives the current. If E is the electromotive force of the battery, and $r$ its internal resistance, the maximum power which is obtainable in an external circuit of equal resistance is—

$$w = \frac{1}{4}\frac{E^2}{r},$$

precisely the same expression as obtained above for alternating currents.

## CHAPTER IV.

### ALTERNATING CURRENT MACHINES.

When discussing the electromotive force of self-induction, we have seen that this is produced by the change in the total induction passing through the electric circuit, or, which comes to the same thing, by the cutting of wires across magnetic lines of force. In a choking coil the magnetization changes, and we thus obtain an electromotive force without the necessity of moving the wire; but when the strength of the field is a constant, and its position in space remains the same, then we must move the wire in order to get an electromotive force, and this is precisely what we do in our alternating current machines or "alternators." The most simple conceivable form of alternator is shown in Fig. 13. Here we make use of the vertical component of the earth's magnetic field, and the electromotive force is a maximum when the wire is either at its highest or lowest position (crank vertical), and when the wire is in the extreme right or left position (crank horizontal) the electromotive force is zero. The apparatus shown in Fig. 13 is, in fact, simply

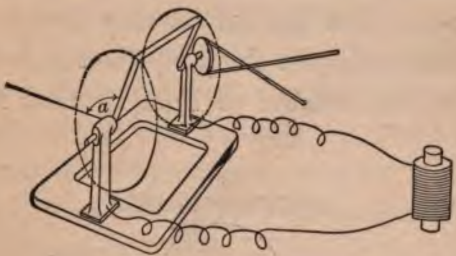



FIG. 13.

a mechanical model of the clock diagram. In this figure the bearings and standards supporting them are represented as forming the terminals; and wires attached to them as shown, and led to a solenoidal electromagnet, will energize the latter. Provided the strength of the field were sufficiently great, we could, when the crank is rapidly rotated by a cord and pulley, produce with the electromagnet the effects I have shown you when the coil was connected to a transformer. But the strength of the field provided by the earth is not nearly sufficient.

We must resort to an artificial field produced by electromagnets, such, for instance, as is shown in Fig. 14, where NS are the polar surfaces of two electromagnets, between which a coil C is rotated. If the polar surfaces extend some distance beyond the diameter D of the coil, we may assume that the field within the space swept by the coil is uniform, and then the number of lines or total induction

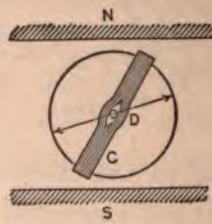



FIG. 14.

passing through the coil at any instant will be proportional to the sine of the angle the coil forms with the vertical. The electromotive force will then be proportional to the sine of the angle the coil forms with the horizontal. Calling H the field intensity, $l$ the length of the coil, $v$ the velocity, and $\tau$ the number of wires counted on both sides, the maximum electromotive force in C.G.S. measure, when the coil is vertical, will be

$$Hvl\tau.$$

It is convenient to introduce instead of H, the number of lines per square centimètre, the total induction F passing through the coil; and instead of the linear velocity, the number of revolutions per second, which, in this case, where we have to do with a two-pole machine, is equal to the frequency $n$. A simple algebraical operation, which need not be reproduced, here gives us the following formula for the maximum electromotive force:—

$$E = 2\pi n F \frac{\tau}{2}.$$

The effective electromotive force is obtained by dividing this expression by the square root of two, and if we wish to get the electromotive force in volts we multiply by 10 to the power of minus 8, or in symbols—

$$e = 2 \cdot 22, \ F\tau n \ 10^{-8} \ldots\ldots\ldots\ldots\ldots (11).$$

Now suppose we remove the armature shown in Fig. 14, and replace it by one wound to give a continuous current. We use the same total length of wire, but spread the turns evenly all round the circle and put on a commutator. As you know, the electromotive force of this machine will now be

$$e = F\tau n\ 10^{-8}.$$

The current flows through the armature in two parallel circuits, and if we allow the same current density in the armature wires, we shall obtain a continuous current of twice the strength of the alternating current. On the other hand, the alternating current will have 2·22 times the voltage, so that the output of the alternator will be 11 per cent. greater than that of the continuous current machine. In the alternator we save the commutator, and you see, therefore, that for equal output the alternator is a cheaper and lighter machine than the continuous current dynamo.

The alternator shown in Fig. 14 is, however, not the kind of machine used in practice. I have only chosen it as a simple example to show the relation between continuous and alternate current machines. In practice the latter are made with a number of poles in order to bring down the speed to reasonable limits, and the wire is not bunched together, but is spread more or less over the surface of the armature. There is also this difference: that the poles of the field surround the armature more closely, and consequently the transition from a north to a south field is more abrupt than in Fig. 14. Notwithstanding these differences, the electromotive force of alternators as practically made is very nearly that given in Formula (11), and the comparison of weight and cost which we found just now holds good for the machines as actually built.

You will see from the diagram on the wall that alternators are all characterized by two main features—a corona or ring of magnet poles and a ring of armature coils, either one or the other being movable. The particular shape of the poles, their arrangement mechanically, the method of winding the armature coils, and many other details, may be changed in many ways, but the main features remain the same.

For purposes of study it is convenient to imagine the pole ring and the armature ring cut open and spread into straight lines. We need then only consider one coil and two or three poles, as shown in Fig. 15. The electromotive force at any moment is obviously proportional to the number of wires which happen at that moment to be covered by one or both poles, care being taken to count, in

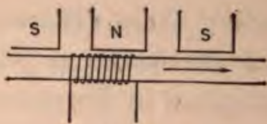



FIG. 15.

the latter case, the difference in the number of wires, since the poles act differentially. We can thus plot a curve giving the resultant electromotive force as a function of the position of the coil in front of the poles, and since at constant speed this position changes proportionately with the time, the curve also gives us the electromotive force as a function of the time. Now you will easily see that the shape of this curve depends on the width of the poles and length of coil. It also depends on their relative shape. But not to complicate our investigation too much, I assume that both poles and coils are rectangular, which in machines of the

Westinghouse and Lowrie Hall type is strictly, and in most other machines is approximately, true. As an extreme case regarding the width of poles, we may take a machine in which the north and south poles are placed so close as almost to touch each other. In this case the width of poles is equal to their distance or pitch. Machines with alternate poles set so closely are not made; but the Mordey, in which poles of the same sign, separated by equal intervals of blank or neutral spaces, is a practical illustration of the same principle. Suppose now that we put into such a field an armature, the whole surface of which is covered by coils, then the length of each coil must also equal the pitch, and there will then be only one position, namely, that in which the centre of the coil coincides with the centre of the pole, when all the wires in the coil are producing electromotive force in the same direction. In every other position the electromotive force in one part of the coil is opposed to that in the other part. In such a machine the wire is not used to the greatest advantage, and the electromotive force curve becomes a zigzag line as shown in Fig. 16. The question which interests us most is that of the effective electromotive force represented by this curve. Experimentally we can, of course, easily determine it. We need only connect a Cardew voltmeter to the terminals of the coil and take a reading; but it is important to know beforehand, that is, before the machine is built, what volts we may expect to get. Consider for a moment what it really is that the voltmeter measures. It is the

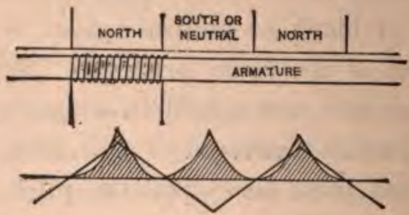



FIG. 16.

amount of heat developed per second in its wire. With the quick alternations produced by the machine there is no time for the wire to change its temperature, and its resistance is therefore constant. The amount of heat dissipated per second is then the square of the effective volts divided by the resistance, and this is also equal to the integral of the square of the instantaneous volts multiplied by the differential of the time, the integration being extended over one second; or we may extend the integration only over the time occupied by one half period and divide the result by that time; this will also give us the heat per second. But as we want to know the volts and not the heat generated per second, we need not concern ourselves about the resistance of the voltmeter wire at all, and simply take the square root of the integral $e^2dt$; this gives the effective volts. To do this graphically, as shown in Fig. 16, scale the ordinates of the zigzag line, square the readings, and plot to an arbitrary scale the result. Thus we obtain the tent-like figure shown in the diagram, the area of which is the integral, or, to speak quite correctly, is proportional to the integral of square of instantaneous volts and time. The height of a rectangle, of equal base and area, is the square of the effective volts. It is thus possible to determine beforehand, for any given arrangement of field poles and armature coils, what the effective volts will be, and, roughly speaking, the larger the shaded area, the higher will be the voltage of the machine. For instance, in Fig. 17, I have assumed that the field of Fig. 16 has been retained, but that the armature coils have been made only half the length for the same number of turns. Only half the surface of

FIG. 17.

the armature is now covered by wire, and the maximum electromotive force is maintained for a quarter period instead of being momentary as before. This gives a trapezoidal line for the electromotive force curve, and the shaded area is now considera-

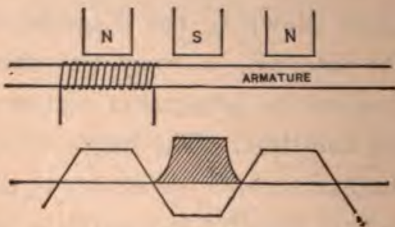



FIG. 18.

bly larger than before. Let us now go back to the first armature, and run it in a field the poles of which are only half the width, the total induction being, however, the same. Here again we get a trapezoidal electromotive force curve (Fig. 18), and the same voltage as in Fig. 17. If we now shorten the coils, we come back to the zigzag lines, but the peaks are higher (Fig. 19), and the voltage, as shown by the shaded area, is again increased. The arrangement shown in Fig. 19 is that usually met with in modern alternators, but owing to the fringe of lines at the corners of the pole pieces, the elec-

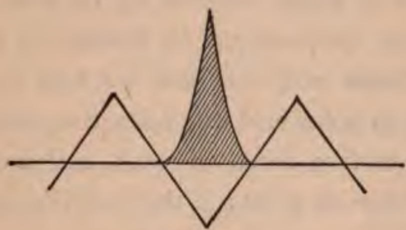


FIG. 19.

tromotive force curve is not quite as sharp as here shown. The peak is rounded off and the sides are more wavy, the curve approaching, in fact, very nearly to a true sine curve. Leaving, however, such refinements aside, it is easy to work out an expression for the effective volts in each given case either graphically by the use of such diagrams as Figs. 16 to 19, or analytically. The operation is somewhat laborious, but in no sense difficult, and it would be useless to burden this lecture with it. I will merely state the results. If you will refer back to Formula (11) you will see that the effective electromotive force of a simple coil, revolving in a uniform field, is given by the product of a constant (in this case 2·22), the total induction or number of lines emanating from one field pole, the number of wires counted on both sides of the coil, and the frequency. If, instead of a machine with two poles and one coil, we had made a machine with 10 poles and five coils, coupled in series, the electromotive force of each coil would have been five times as great; if the machine at 20 poles and 10 coils it would have been 10 times as great, and so on. You see, therefore, that the electromotive force is simply proportional to the total number of wires coupled in series, and to the number of pairs of poles, and Formula (11) is right for a machine with any number of poles. The same kind of formula is also correct for any arrangement of poles and coils, but the coefficient is different in each case. This coefficient is really the ratio of 'he electromotive force of the alternator to that of a continuous current dynamo of equal weight and arrangement of field and armature. If the machine has $r$ pairs of poles, and runs at a speed of N revolutions per minute, its electromotive force of a continuous current machine is

$$e = p\mathrm{F}\tau \frac{\mathrm{N}}{60} 10^{-8} \quad \dots\dots\dots\dots\dots\dots (12),$$

or if by Z we denote field strength in English measure—

$$e = p\mathrm{Z}\tau\mathrm{N}10^{-6} \quad \dots\dots\dots\dots\dots\dots (13).$$

Let K be the coefficient which depends on the shape of poles and armature coils, then the electromotive force of our alternator is

$$e = \mathrm{K}p\mathrm{F}\tau\frac{\mathrm{N}}{60}\ 10^{-8} \quad \dots\dots\dots\dots\dots\dots (14).$$

$$e = \mathrm{K}p\mathrm{Z}\tau\mathrm{N}10^{-6} \quad \dots\dots\dots\dots\dots\dots (15).$$

$$e = \mathrm{KF}\tau n 10^{-8} \quad \dots\dots\dots\dots\dots\dots (16).$$

To find the electromotive force we must therefore determine the coefficient K for each case, and, as I have already said, this is not a difficult mathematical problem. The result for the cases I have brought before you is as follows:—

(1). If machine gives a strictly sinusoidal electromotive force, . $k = 2{\cdot}22$

(2). Width of poles equal to pitch, and length of coils equal to pitch, . $= 1{\cdot}160$

(3). Width of poles equal to pitch, and length of coils equal to half the pitch, . . . . . . $= 1{\cdot}635$

(4). Width of poles equal to half the pitch, and length of coils equal to pitch, . . . . . . $= 1{\cdot}635$

(5). Width of poles equal to half the pitch, and length of coils equal to half the pitch, . . . $= 2{\cdot}300$

If you compare the first and last line of this table, you will find that there is only 3½ per cent. difference between the two coefficients. The last line refers to machines as actually built, and the first line to ideal machines having a true sinusoidal electromotive force curve. You see that, as far as the effective electromotive force is concerned, the assumption that ordinary commercial alternators follow the sine law is practically correct.

## CHAPTER V.

### MECHANICAL CONSTRUCTION OF ALTERNATORS.

Speaking generally, we may say that the constructive requirements and the points to which particular attention must be paid in designing alternators are very much the same as obtain in dynamos, but there may be certain differences. In the first place, the armature of a dynamo is, on account of its commutator and brushes, necessarily more complicated than that of an alternator. On the other hand, the field is simpler. The majority of dynamos are made for low or moderate voltage, whilst alternators are generally made for high voltage. This requires greater care in the insulation, and compels us to avoid certain methods of winding, which for a 100-volt dynamo are quite admissible. In the design of both kinds of machine we must pay attention to eddy currents and hysteresis, but in alternators these disturbing and injurious effects are far more serious than in dynamos. The reason is that both the wire and the iron, if the armature has an iron core, are subjected to a more rapid reversal of induction. Special precautions must therefore be adopted. The core must be well laminated, and the conductor should not exceed a certain section. What that maximum section should be depends, of course, on the general design of the alternator, but we may take it roughly that, where round wire is used, its diameter should not exceed 140 mils., and where strip is used its thickness should not be more than 100 mils. Another and very effectual cure for eddy currents is to embed the conductor entirely in iron, an arrangement which has been first proposed by Wenstrom, and has been largely used by Brown, the latest example being his large three-phase current alternator at Lauffen. The conductor is a solid copper rod of about 1½ inches in diameter, threaded through holes in the armature core. A conductor of that size, if placed on the surface of an armature where it is subjected to some 80 field reversals per second, would get hot in a few minutes, yet, arranged as it is in Mr. Brown's "three-phaser," it keeps perfectly cool. It is the fact that the conductor is surrounded on all sides by iron which produces this result. A still more striking illustration of the effect of iron in preventing eddy currents is Thomson's welding machine. Here we have a solid conductor of many square inches in area, in which the welding current is generated. But this conductor is the secondary circuit of a transformer, and is surrounded by the iron of the transformer. Professor Thomson's explanation of the fact that in all such cases eddy currents are avoided is that the speed at which the wire cuts through the lines of force is much greater than its speed of motion, that, in fact, the lines at first yield and, so to say, stretch, but finally, when the tension becomes too great, snap suddenly past the wire. Thus all parts of the wire are cut at almost the same instant by the lines of force, and this leaves no time during which the differences of electromotive force, and, therefore, eddy currents, could be developed in the wire. I, personally, do not feel competent to either confirm or refute this explanation, but coming from so high an authority as Professor Elihu Thomson, am satisfied to accept it. That wires embedded in iron, or surrounded on all sides by iron, are nearly free from eddy currents is, however, an undoubted fact.

From what I have here said you will see that in one way or another we can avoid, or at least greatly reduce, the loss of power by eddy currents in alternators. Now let us see whether this is also the case with the other source of loss, namely, "hysteresis." Under this term we comprise a certain phenomenon, first investigated by Ewing, and which may be popularly described as "magnetic friction." The lines of force in being forcibly dragged through the iron core of the armature continually change its magnetization, and the core, even if most carefully laminated, so as to avoid eddy currents, still becomes hot if revolved in an excited field. It is at once obvious that the power thus wasted will be the greater the more rapid the reversal of magnetization, and the greater its amount. This loss takes place in dynamos as well as in alternators, but to a different extent. In a dynamo the reversal is comparatively slow. Take, for instance, a two-pole machine, running at 600 revolutions per minute, or 10 revolutions per second. The whole mass of the armature core undergoes, therefore, in every second ten complete cycles of magnetic change. But in modern alternators the change is about ten times as rapid, the frequency being 100. If we allowed the same induction, that is, number of lines per square centimètre of core section, the alternator would waste ten times the power, and this would, of course, be inadmissible. There is only one way in which we can reduce the waste of power, and that is by adopting a lower induction. Thus, whilst in dynamos the induction ranges from 14,000 to 20,000 lines per square centimètre, it is only

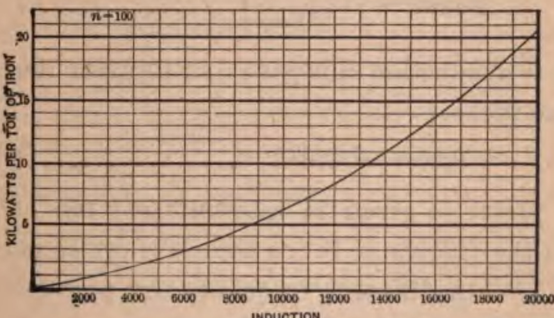



FIG. 20.

about 5,000 in alternators. The exact induction at which it is best to work varies, of course, with the type and size of machine, and as every design is a compromise, you must not consider the 5,000 as a hard-and-fast rule. To enable you, however, to deal with each given case on its own merits, I give in Fig. 20 a curve showing the loss of power by hysteresis per ton of iron when the frequency is 100. The induction is measured on the horizontal, and the power (in kilowatts) on the vertical. This curve has been compiled from the experimental results of Professor Ewing. I may incidentally mention that this curve is approximately represented by the equation—*

$$\text{Power} = 180 \left(\frac{B}{1000}\right)^{1.65} \text{..............(17)},$$

or if the induction B is given in English lines per square inch—

$$\text{Power} = \cdot 160\ B^{1.65} \text{................(18)}.$$

The power is given in kilowatts per ton of iron when the frequency is 100 complete cycles per second. For a different frequency the power is proportionally altered.

* Steinmetz in his classical researches on hysteresis gives the exponent as 1.6.

There being necessarily always some waste of power, if the armature has an iron core it was natural that inventors should turn their attention to the construction of an alternator with a coreless armature. In fact, the Meritens machine, which was one of the first commercial alternators, and is used to this day in lighthouse work, has no iron in the armature. Then there comes the Siemens, also without iron, the Ferranti, and the Mordey. In all these machines the loss by hysteresis is avoided, and if this were the only consideration, they would undoubtedly be better than their rivals with iron-cored armatures. But as I have said before, every design is a compromise, and it is quite possible that the machine with iron in its armature is as good a compromise as one without iron. The fact that the majority of American machines, all the German machines, including those now made by Messrs. Siemens and Halske, and a good half of the English machines have iron-cored armatures, is in itself sufficient proof that the hysteresis loss is not an insurmountable obstacle. There are especially two points in favour of using iron. The first is that we are thereby enabled to give the armature greater mechanical strength than can be done in machines where the armature coils are attached singly and held by insulating material. The second is that the presence of iron tends to diminish the magnetic resistance of the air-gap, and thus saves exciting energy. In Mr. Brown's three-phaser, for instance, the total exciting energy does not amount to more than ½ per cent. of the total power.

I have, a moment ago, spoken of the differences between alternators and dynamos from an electrical and mechanical point of view. There remains yet to notice an important point of difference, namely, the absence in alternators of a commutator and brushes. You all know that these are the most delicate parts of a dynamo, and although in modern machines of moderate voltage these parts are perfectly reliable and easily handled, the case is different when we attempt to build dynamos for 1,000 or 2,000 or more volts. We encounter then difficulties which are absent from alternators, and it is mainly on this account that engineers who have to design power transmission schemes over long distances are beginning to turn their attention to some forms of alternator as the most certain means of solving such problems. I shall have something more to say on this subject in the third lecture. For the present I must limit my remarks to the machines as required for lighting.

## CHAPTER VI.

### DESCRIPTION OF SOME ALTERNATORS.

In the limited time at my disposal it would be impossible for me to give you anything like an exhaustive account of the various machines now in use. I shall therefore only describe a few of them as being representative examples.

1. *The Ferranti Alternator.*—The field magnets are wrought-iron bars of trapezoidal section (Fig.

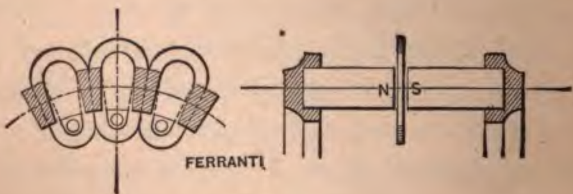



FIG. 21.

21), cast into massive yoke rings, which can be drawn apart at right angles to the shaft, so as to expose the armature for examination and repair.

The latter is of disc pattern, and the coils are inserted in pairs. The conductor is a corrugated copper strip wound with a strip of vulcanized fibre of equal width upon a laminated brass core. The conductor is thus insulated from the core, and the latter is insulated from the supporting ring. This double insulation is an important feature of the machine. The core is held in gun-metal cheeks, which are provided with side wings for ventilation. The attachment of each pair of cheeks to the supporting ring is by means of a shank passing through insulating washers into a cavity in the ring, and secured by a nut. The cavity is cast out with sulphur. To avoid too great a loss by eddy currents the conductor is made very thin; the winding is split up into two, four, or more parallel circuits. I may here incidentally mention that where an armature winding is thus split up, great care must be taken to have all the magnets of equal strength, as otherwise there would be created with the armature differential currents, which would waste far more power than the eddy currents, which the arrangement was intended to avoid. The Ferranti machines now working at Deptford are giving an electromotive force of 10,000 volts, and to prevent flashing over to the magnets the latter are provided with double caps of ebonite.

2. *The Mordey Alternator.*—This is also a coreless machine of the disc pattern, but the armature is fixed whilst the magnets revolve. The armature coils (Fig. 22) are wedge-shaped, and the conductor

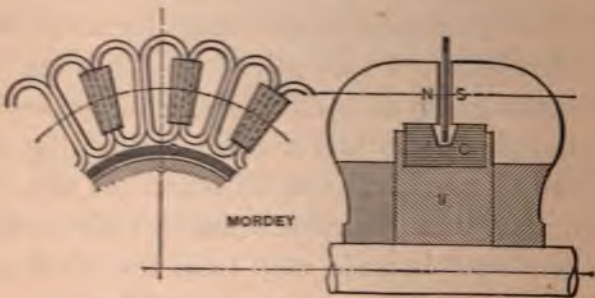



FIG. 22.

is a thin copper strip wound on a slate core, the layers being separated, as in the Ferranti coils, by a thin strip of insulating material. The attachment is made at the outer and wider end of the coil to a gun-metal supporting ring. The magnets are of cast-iron, and so shaped as to require only one coil C of exciting wire. This is wound on a central cylindrical part $y$, to both sides of which are pole pieces of peculiar star-like form. Thus the poles on one side of the armature are all of the same sign, and those on the other side are of the opposite sign, the lines of force passing from N to S at right angles through the surface of the armature, and all in the same direction. There is thus, properly speaking, no reversal of magnetization, but merely a change from full induction when a wire is between opposite poles, to no induction when it is between neighbouring poles, and the general effect is the same as if we had half the field strength alternating. To apply our formulæ for the electromotive force of this machine we must, therefore, introduce not the whole field strength F or Z, but half its real value.

3. *The Westinghouse Alternator.*—As a good example of an alternator, the armature of which contains iron, we may take the Westinghouse machine, which, in its important details, is very similar to the Thomson-Houston. The armature is cylindrical (Fig. 23), and is covered by link-shaped coils, with the wires parallel to the shaft, the rounded ends of the coils C being bent inwards, and secured

to the end faces of the armature core. In the Thomson-Houston machines the coil ends are not

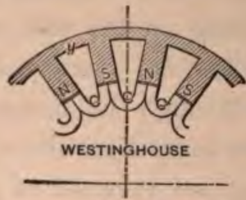



FIG. 23.

turned inwards. The field magnets NS are set radially outside the armature, and their outer ends are connected by a yoke ring Y. According to our theory, the best arrangement as regards width of coil is half the pitch, which means that the central space of the coil should have the same width as the magnet, but Professor Thomson, when experimenting with various coils, found that a coil having a slightly smaller internal space gave a higher electromotive force when the machine was working under full load. His explanation is that the current in the armature wires alters the original magnetization of the field, tending to concentrate the lines towards the leaving edge of the pole piece, and thus produces a more intense but narrower field. The inner space of the coils, which is free from winding, should therefore also be made narrower.

4. *The Kapp Alternator.*—In this machine (Fig. 24) the armature is of the disc pattern, and con-

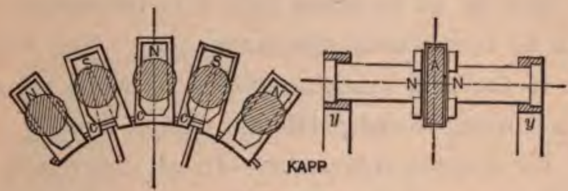



FIG. 24.

tains an iron core A made by coiling a strip of thin charcoal-iron with a strip of paper upon a supporting ring. The coils are wound transversely round the core. The field magnets in the larger machines are of wrought iron, with expanded pole shoes, and are set parallel to the shaft on both sides of the armature core, presenting the same poles NN SS on opposite sides. The outer ends of the magnets are joined by cast-iron yokes YY. Owing to the angular position of the pole shoes, each wire does not enter the field simultaneously over its whole length, but the entry is a little more gradual, whereby the sharp peaks in the line of electromotive force (Fig. 19) are toned down, and the curve is made to approach a sinusoidal form. The current is collected by rubbing contacts from insulated rings, which are set on opposite sides of the armature, that is, so far apart that it is impossible for a man to touch both simultaneously. The coefficient K for this machine varies between 2·3 and 2·7, according to the particular design chosen.

5. *The Kingdon Alternator.*—In all the machines described up to here, the wire, either on the field or on the armature, is in motion, but in the Kingdon machine all the wires are at rest, the only revolving part being an armature containing no wire. The machine consists of a laminated iron cylinder, with radial teeth projecting inwards, and the armature and field coils are wound over alternate teeth. The revolving part is a wheel provided with half as many laminated iron keepers as there are teeth in the stationary part, and these keepers are so arranged as to bridge magnetically neighbouring teeth. Thus the teeth over which the armature coils are wound become alternately parts of a posi-

tive and negative magnetic circuit, and an alternating current is produced.

6. *The Kennedy Alternator.*—Mr. Kennedy has further developed this idea, mainly by reducing the number of armature and field coils, and avoiding the generation of an alternating current in the lat-

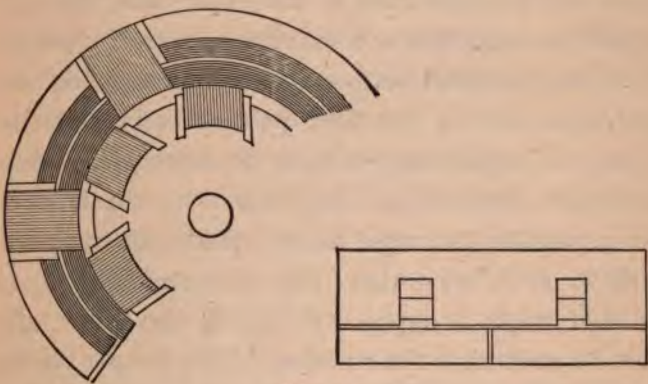


FIG. 25.

ter. The machine (Fig. 25) has two armature and two field coils wound in pairs, and placed into recesses in a skeleton frame of soft iron laminated bars. There are two keeper wheels on the spindle, but stepped in relation to each other by half a period, so that when the keepers of one wheel have completely closed the magnetic circuits around one pair of field and armature coils, the keepers of the other wheel are midway between the fixed bars, and the magnetic circuits around the second pair of armature and field coils are interrupted. The electromotive force in those coils is at that moment zero, and it is also zero in the other coils, through which the induction is a maximum. Half a period later the induction becomes a maximum in the second, and zero in the first pair of coils, and this change of induction produces an alternating electromotive force in all the coils. Now it is obviously possible to couple the two field coils in series, and in such way that the electromotive force created in one is opposed to that in the other, and thus to neutralize the reaction of the keeper on the exciting circuit. The exciting dynamo has then merely to overcome the ohmic resistance of the two coils, as in any other machine. The two armature coils may be coupled in series or parallel.

## CHAPTER VII.

### TRANSFORMERS.

I have already drawn your attention to the fact that alternators are generally designed for high voltage. The reason is obvious. If we wish to carry the current, be it for lighting or power, to any distance, we must use a high voltage, in order to bring the section of our conducting wires or mains down to a size which makes the whole enterprise commercially possible. But to give our customers a current of some thousands of volts would be dangerous and inconvenient, for glow lamps require a current of about 100 volts when arranged in parallel, that is, in the way in which they are of most use to private consumers. The question therefore arises what to do with our high-pressure current when we have brought it to the place where its energy is required for lighting lamps. Obviously we must transform it; we must lower its voltage and increase its strength. Now there are two ways in which this may be done. We may use the current to work a motor, and use the power given out by the motor to drive a dynamo of 100 or 200 volts. The direct current can then be distributed to the lamps in the usual way, and we may even supplement the installation by secondary batteries, so as to be able to shut down our machinery during the hours of minimum demand. As far as I know, this system of transformation has only, up to now, been used in one installation, namely, at Cassel, in Germany. At the first glance it may seem complicated and costly, but it has many advantages, which will probably lead to its adoption in other towns.

The other system which is at present in general use is that of direct transformation by means of induction coils. Here we need no moving machinery, but simply a stationary apparatus consisting of a laminated iron core and two coils (Fig. 26). One of these consists of many turns of fine wire, and is technically termed the primary coil P, and the other of fewer turns of stouter wire called the *secondary* coil S. The high-pressure current is brought by the mains *w*, and the low-pressure current is supplied to the lamp L by the secondary mains W. You will observe that there is absolutely no connection between the two sets of mains, and this is a great guarantee for the safety of the system. The action of this apparatus, which is technically known under the name of "trans-

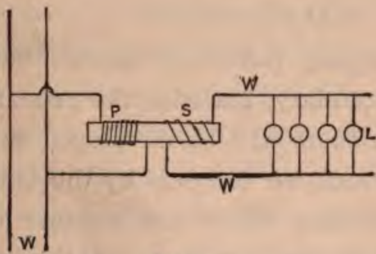



FIG. 26.

former," will be clear to you from what I have said in the first lecture about the generation of an alternating electromotive force. The primary coil magnetizes the iron core in alternate directions, and at each reversal the lines of force cut through the wires of the secondary coil. The latter must therefore become the seat of an alternating electromotive force. If we denote by F the total induction, and call $n$ the frequency, the maximum electromotive force generated in each turn of wire is

$$2\pi n F,$$

and the effective electromotive force is this value divided by the square root of 2. If the coil contains $\tau_2$ turns, the total effective electromotive force in the large circuit will be

$$e_2 = 4{\cdot}45\, n F \tau_2\, 10^{-8} \text{ volts} \quad \ldots\ldots\ldots\ldots (19).$$

The changing induction affects, however, not only the secondary, but also the primary coil, and in the latter there will be developed an electromotive force which we compute by the same formula, only substituting for $\tau_2$ the number of primary turns $\tau_1$. You see, therefore, that the transforming ratio is given by the ratio of the turns of wire in each coil; but this is only approximate, since not the whole electromotive force generated in the secondary reaches its terminals. We must deduct the electromotive force used up in overcoming the ohmic resistance of the secondary coil. In like manner the electromotive force which opposes the current in the primary coil is a little smaller than the terminal electromotive force, because the ohmic resistance also opposes the primary current. The transforming ratio therefore varies with the load, but I may at once say that in good transformers the variation as determined by ohmic resistance is exceedingly small, generally about 2 per cent. There is, however, another cause of variation, namely, magnetic leakage, and a transformer made as shown in Fig. 26 would exhibit this phenomenon in a most objectionable degree. You see that the two coils meet in the middle of the core. Now the primary wants to magnetize the core in one direction and the secondary wants to magnetize it in the opposite direction. The result is that the two streams of induction come, so to speak, into collision about the middle of the core, and some of the lines which the primary coil tries to shoot through the secondary are squeezed out sidewise, and contribute nothing to the secondary electromotive force. You might perhaps think that this is of no moment, for we can make up for the loss of these lines by putting a few more turns of wire on the secondary. But if we did that we should get too much electromotive force at light loads. To see this clearly let us begin with no load on the secondary. Then there is no current in S and no collision. The lines created by the primary pass without opposition through the secondary, and F in Formula (19) has its full value. Now switch on some lamps and a secondary current will flow. We shall have some collision of lines, and F in the formula, and therefore the electromotive force, will become smaller. The more lamps you switch on, the more current flows through both coils, and the more violent becomes the collision, and therefore the number of lines lost. In a word, we generate more lines than we can utilize. The obvious remedy for this defect is to place the coils relatively to each other into such a position that the lines generated by the primary cannot evade passing through the secondary. For instance, we can wind one coil over the other, or we can split up the coils into short sections and place them alternately over the core. Even with these precautions there is some magnetic leakage, but this does not as a rule lower the voltage by more than 1 or 2 per cent. Thus in a good transformer we may expect to get, with constant primary voltage, a terminal pressure varying *between* 102 and 99 or 98 volts, when the load is increased from zero to full out-put. These figures refer to small transformers of 50 or 100 lamps. With large transformers it is quite possible to limit the total voltage drop to something under 2 per cent. The transformer shown in Fig. 26 is defective in other ways besides its great voltage drop. The lines passing through the core have to come back through air, and the great magnetic resistance of their path through air requires a strong magnetizing current, or, in other words, the primary current will be considerably greater than in a transformer, in which the return path is made more easy. One way of doing this is to increase the surface of the core ends, and this Mr. Swinburne has done in his "Hedgehog" transformer. The core consists of iron wires which at the ends are curved outwards. Thus part of the return path is through iron. Another method, and this is generally used, is to make the whole return path of iron. We may, for instance, employ a closed iron frame (J, Fig. 27), and wind the primary and secondary coils C over each other on two opposite sides of this frame. The iron frame or core is composed of thin plates, more or less insulated from each other to avoid eddy currents. This type of transformer is called a "core transformer." Or we may employ only one coil and surround it by a double

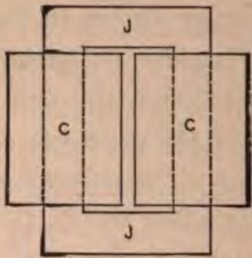



FIG. 27.

frame, as in Fig. 28, a kind of iron shell, and this construction is called a "shell transformer." Both figures have been drawn to represent transformers of equal out-put. The depth of the core is supposed to be equal, and its width in Fig. 28 is twice as great as in Fig. 27, to make up for their being only one coil. At the first glance it is difficult to say which is the better transformer, though practically the balance of advantages seems to lie with the shell type, which is most in favour with the makers of this kind of apparatus. If we enquire what it is we must aim at in the design of a good transformer, we find that the length of wire should be small in order to reduce ohmic resistance and cost, that there should be as little iron as possible, and that the magnetic circuit should be short. Now these are contradictory conditions. To reduce the length of wire we must work with a high total induction, so that a small number of turns should give us the required electromotive force. But a large induction means either a great loss by hysteresis or a stout core, and a stout core means that the length of each turn of wire is great. It further means a longer magnetic circuit and a greater

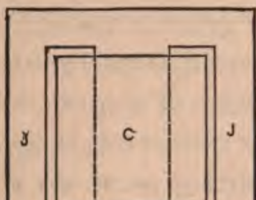



FIG. 28.

weight of iron, which again increases the loss by hysteresis. You see here again the successful design must be a compromise, but a compromise in which a preponderance of weight is given to hysteresis. We must remember that a transformer is continuously at work whether we take current from it or not, and hence the hysteresis loss goes on day and night. Even an extra loss of ½ per cent. by hysteresis will therefore be felt in the all-day efficiency of the apparatus, and is, therefore, more serious from an economical point of view than a loss of several per cent. by copper resistance, because transformers, when used for lighting, only work a very short time daily under full load. On the other hand, we must not allow an excessive copper loss, as this would disqualify the transformer on account of too great a voltage drop. We are thus hemmed in on all sides by conflicting conditions, and the design of a good transformer is by no means so easy a task as might appear at the first glance. As a starting point, we may take it that the magnetic and the electric circuit should be as short as possible, and this condition will be best fulfilled by a circular or square shape, or as near an approach to such a shape as possible. In fact, if you examine the successful transformers in the market, you will find that this condition is fulfilled in either one or the other circuit, but not in both. I have not succeeded in establishing, and I can therefore not give you any, hard-and-fast rules for the construction of transformers, but in order to enable you to see what enormous influence slight alterations in the proportions have on the weight and cost of the apparatus, I have prepared for your proceedings some 27 different designs, all for a 100-light transformer. Four of these designs are given full size in Plates I. and II., pages 156–7. In all these the copper loss is 2 per cent. The hysteresis loss is given in each case. You can see at a glance what a great difference there is in the amount of copper required, and how by a skilful choice of the proportions the cost of the apparatus can be reduced without lowering its efficiency.

Before concluding this part of my subject, I wish to draw your attention to the relation existing between the linear dimensions of a transformer and its out-put and hysteresis loss. Imagine that after designing a score or so of transformers you have at last arrived at a type with which you feel satisfied; but suppose it is not the size you want. How will you alter its linear dimensions? Let us try what we shall get if we make everything twice as big, including the size of the wire. We retain the induction per square centimètre at the 4,000 or 5,000, which we found will give us a loss of say 1½ per cent. by hysteresis. We also retain the number of turns in both coils. The total induction is then four times as great, and the electromotive force also four times as great. The resistance of the coils has been reduced to one-half its former value (area of wire four times as great, and length twice as great). If we are satisfied to have the same copper loss, we can allow a current which will give us four times the previous voltage loss, but as the resistance has been halved the current will be eight times its former value. Thus the current is eight times and the volts are four times as great as before; the out-put will, therefore, be 32 times as great. But 32 is two to the fifth power, and hence we see that the out-put of a transformer varies as the fifth power of its linear dimensions. The weight, cost, and hysteresis loss, on the other hand, all vary as the cube of the linear dimensions, and the weight per kilowatt of out-put varies inversely as the square of the linear dimensions. Or, in figures, if 40 lbs. of copper and iron were required for each kilowatt produced by the small transformer, only 10 lbs. per kilowatt will be required in the larger; and if the small transformer wasted 2 per cent. of its power in hysteresis, the large transformer will only waste ½ per cent. Let the large transformer have $x$ times the out-put of the small one, then linear dimensions must be proportional to $x^{\frac{1}{5}}$. Weight and cost per kilowatt and percentage loss by hysteresis will be proportional to $x^{\frac{2}{5}}$. This calculation neglects, however, the working temperature which, for obvious reasons, must not exceed a certain limit. In practice it is found that for every watt lost by hysteresis, eddy currents, and ohmic resistance a cooling surface of from five to ten square inches must be provided. As the larger transformer has, relatively to its out-put, a smaller external surface than the smaller transformer, it is not possible to take full advantage of the law between linear dimensions and out-put here given. In part, owing to the difficulty of heating, we very soon arrive at a limit of out-put when a further increase of dimension varies the out-put scarcely faster than the weight and cost. Thus in practice it is found that although a transformer of 12 kilowatts does not weigh nearly as much as four transformers of 3 kilowatts, a transformer of 60 kilowatts weighs and costs very nearly twice as much as a transformer for 30 kilowatts.

# CHAPTER VIII.

## CENTRAL STATIONS AND DISTRIBUTION OF ALTERNATING CURRENTS.

The principal reason for the use of alternating currents in connection with the supply of light from a central station to private customers is that in consequence of the high pressure which can safely be used we are able to take on customers, whether near or far, and thus carry on the business of light purveyors on a larger scale, and presumably with a greater profit, than if we were restricted to those customers only who live near the station. In a general sense this argument is perfectly sound, but it would be a mistake to apply it indiscriminately and say that in all cases the supply by means of alternating currents is preferable to that by continuous currents. Whether an engineer has to design works himself, or merely to inspect and approve works carried out by others, it will always be his first concern to see that the works shall be a commercial success. We cannot build central stations or any other works without the aid of the financier, and the financier cares very little for any technical perfection; all he cares for is that the work should pay, and unless the engineer can give him that assurance he will not co-operate. Hence it is the business of the engineer not only to design his works so as to be technically a success, but also commercially.

In considering the relative merits of the two systems, we must take into account a variety of local circumstances, some of which not only are beyond the reach of mathematical representation, that is, representation by concrete figures which we can use in our calculation, but may even be but vaguely known at the time the station is being designed. For instance, the number of lamps which will be required in any given district, the daily lighting time of each lamp, and the distribution of lamps between the different classes of houses in the district, are matters which we cannot foretell with absolute certainty. We can but make a guess based on previous experience. Another matter of some importance, but about which it is extremely difficult to form an estimate beforehand, is the danger of being served with an injunction for noise or vibration by some of the kind neighbours, who are always on the look-out how to make a little money out of the difficulty of others. This danger is evidently greater in the direct current system, because with it we have not a very wide choice as to the position of our station, but must place it fairly near to and preferably in the centre of the district to be lighted. With the alternating current system we can afford to go farther afield with our station, into a neighbourhood the inhabitants of which are not so particular as to noise and vibration. Then there is the question of the total extent of the district to be lighted, the possibility of working by water power, or if not, the cost of coal and water, the quality of the latter, the possibility of obtaining condensing water, and many other points which have to be considered.

If we have to do with a compact and densely lighted district, where most of the lamps can be placed within a few hundred yards of the station (or at any rate within a radius of about 1,000 yards), then the direct current system is generally the best. One of its greatest advantages lies in the fact that we can supplement the dynamos by storage batteries, and use the latter during the hours of minimum demand. For economical reasons we are obliged to use compound engines, but, as you know, a compound engine, except when condensing, does not work with economy when lightly loaded, and it is therefore advantageous to shut down the engines altogether in the early hours of the morning and during the daytime, putting the batteries on for the supply of the few lamps required. In this respect the direct current has a distinct advantage, but this advantage becomes less and less felt as the total power of the station is increased, because in a large station the number of lamps, even in the daytime, will be large enough to fairly load a small engine, and if we can obtain condensing water the engine, even if only partly loaded, will work with fair economy.

A point at present in favour of direct currents is the ease with which they can be used for motive power, but there is every prospect that ere long alternate current motors will become a practical success. At any rate, the use of motors on town circuits has with us not yet become so popular that we need attach any great weight to this point. The principal advantage of the alternating current system is that we can use small mains, and yet keep the pressure throughout the district very nearly constant. With continuous currents not only do we require more copper in the mains and feeders, but where the feeders are long, the loss of pressure in them amounts sometimes to as much as 20 per cent. of the total or station voltage, and in such cases some complicated arrangements are required for the regulation of the voltage, so as to keep the pressure at the feeding centres at least approximately constant.

There are two ways in which we can use transformers. We can bring the high-pressure mains right into the house of each customer and give him his own little transformer, or we can place large transformers at certain sub-stations, and lay through the streets a second system of low-pressure mains, with house connections, in the same way as if the supply were by direct currents, only that in this case the low-pressure mains need not be so large, since we can put down as many sub-stations as we please, and thus reduce the distance to the lamps to any desired limit. The system of a separate transformer for each customer has hitherto been most used, but it is not the best. It is true that by it we save the cost of the secondary mains and the cost of the sub-stations, items which a company in its early pioneering days, when customers were few and far between, could not easily afford. On the other hand, the objections to the use of separate transformers are great, and as time goes on, that is to say, as the use of the electric light extends, these objections acquire additional weight. In the first place, there is some danger in having a high-pressure apparatus in one's house. You may put your transformer into the cellar in a fireproof case and lock it up, but when you have thousands of transformers in as many houses, the chances are that in one or two cases the locking up may be forgotten, and some inquisitive person may touch a terminal. A further objection lies in this: that a number of small transformers cost more money and waste more energy than one large transformer. Let us take for example twenty houses, each wired for fifty lamps. Each house must get its 50-light transformer. The whole of the fifty lamps will not be lighted simultaneously every day. Probably not more than half-a-dozen times in the year will each transformer be worked at its full out-put, and there is the hysteresis loss going on in it day and night. This loss means waste of power and development of heat; indeed, I have heard of one case in which the heat given out by a transformer placed in a wine cellar was sufficient to keep the cellar at a nice even temperature all the year round. General experience tells us that scarcely more than ½, or at most 60 per cent., of the lamps wired in a district are ever alight simultaneously. The maximum joint demand for current of our twenty houses will therefore never exceed 600 lamps, and we can substitute for the twenty separate transformers of fifty lights each, one single transformer of 600 lights. From what I have said before about the influence of size on the cost of transformers, you will see that the single large transformer will cost scarcely more than a third the money required for the twenty small ones, and that even if we put down two large transformers so as to keep one in reserve, we shall do it for little more than half the money. Similarly, the loss of power by hysteresis will be reduced to one quarter, and this is a very important consideration. Take for instance a station designed for 20,000 lamps, of which 12,000 will be alight simultaneously during the two or three hours of maximum demand. The average lighting time of each lamp fixed is in London about 500 hours per annum. If we allow with small transformers a loss of 2 per cent. by hysteresis, the power continuously absorbed by all the transformers connected to the central station will be equivalent to that required by 400 lamps. We are wasting, therefore, day and night current which could feed 400 lamps. In a year we waste not less than 3,500,000 lamp hours, whereas our total income from the 20,000 lamps is only 10,000,000 lamp hours. This means that even if there were no other sources of loss we would have to send out energy from our station representing 13,500,000 lamp hours, but we could only get paid for 10,000,000 lamp hours. This is only 74 per cent. efficiency. Now suppose we use sub-stations and large transformers, the hysteresis loss will fall to

1 per cent., and the efficiency will rise to over 90 per cent. We can further improve the efficiency by putting down at the sub-station not one transformer only, but two or more of different size, and make arrangements for the insertion or withdrawal of transformers from the two circuits (the high and low-pressure circuits), in accordance with the demand for current, so that during the hours of light load the hysteresis loss will only take place in the smallest transformer of the group. Mr. Ferranti, Mr. Gordon and myself have, independently of each other, devised an apparatus which switches in and takes out the transformers automatically.

The employment of large transformers at sub-stations has this further advantage: that the total length of high-pressure mains is thereby considerably reduced, and that there are no branch connections on these mains. We are thus able to get higher insulation. You know that an insulation of many hundreds of megohms per mile can be easily attained in a continuous cable, but after the cable has been laid, and branch connections have been made, the insulation is much lower, the reason being that at every joint the insulation has first to be stripped, and then made good again. Now it is one thing to put on the insulation in the factory, where every precaution can easily be taken to ensure perfect work, and it is quite another thing to do the same kind of work at the bottom of a trench or pit in the street. No matter how careful we are, the insulation put on under these circumstances can never be so good as that put on by the covering machines in the cable factory. For this reason a system of simple mains radiating from the central stations to the sub-stations must show a higher insulation than a complicated network of mains covering the whole district.

I have given you here the main reasons for the adoption of transforming sub-stations in connection with alternate current distribution, but, in applying them to each given case, you must not forget to take into account the commercial element. A system of working may be scientifically the best, and yet not the best financially. Thus the system generally applied at the present time in London in alternating current stations is that of a separate transformer for every customer, not because it is theoretically the best, but simply because it is commercially the only feasible system. I have, however, no doubt that as the use of the light becomes more general, the various companies will find it advantageous to change to the system of sub-stations. I have hitherto not said anything as to the comparative cost of continuous and alternate current stations, and it is indeed very difficult to state it in any definite way. The cost of boilers, engines, and accessory apparatus will be about the same in both systems. The alternators will be a little cheaper than the dynamos, and there will also be the cost of the battery against the continuous current system. On the other hand, we have to remember that the whole engine power may be slightly less, since during the hours of heavy lighting the battery assists the engines. Taking, then, one thing with another, there will not be any very great difference in the cost of the plant at the central station on the two systems. The difference is mostly outside. If the district is large, the extra cost of the heavy feeders and mains will, with continuous currents, be much greater than that of the high-pressure feeders and low-pressure mains if alternating currents are used, and the margin left will be more than sufficient to pay for the transformers at the sub-stations. If the district be small, and the lighting compact, then the balance will be the other way. The cost of mains will be about the same in the two systems, but we shall not be able to save enough on the cost of the feeders to pay for the cost of the transformers. Each case must, in fact, be judged on its own merits, and what I have said here about comparative cost is merely intended as a guide in forming such a judgment.

## CHAPTER IX.

### EXAMPLES OF CENTRAL STATIONS.

As examples to illustrate this lecture, I choose three types of central stations, distinguished from each other mainly by the character of the motive power employed. In the first type the motive power is steam, in the second it is water power, and in the third it is electricity.

1. *The Sardinia Street Station of the Metropolitan Electric Supply Company.*—The boilers are of the Babcock Wilcox type, and placed on the ground-floor. The battery of boilers is parallel to the two rows of engines in the adjoining room, also on the ground-floor, but at a slightly higher level. This arrangement of boilers and engines has the great advantage of reducing the length of steam-piping, and minimizing the inconvenience resulting from a failure of any particular length of steam-pipe. The steam-pipe forms what is technically termed

a ring main, and valves are inserted at suitable points, so that any length can be cut out without disturbing the supply of steam through the rest of the piping. Adjoining the boiler-room, and connected with it by a tram line, is a vast underground coal store; a very admirable arrangement, especially in a station situated, as is that of Sardinia Street, in a district where coals can only be delivered by cart, and where, consequently, the delivery may, in times of heavy frost or fog, be interrupted for some days or weeks. The engines are of the compound high speed Westinghouse type, and drive by belt Westinghouse alternators placed on an upper floor. Alongside one wall of the machine-room is placed the switchboard, by means of which any desired combination between the alternators and external circuits can be quickly made. During the hours of light load all the circuits are put on to one or two machines, but as the load increases other machines are started, and some of the circuits are transferred to them. The machines are not worked in parallel. As regards the mains, I must mention an ingenious arrangement due to Mr. Bailey, the engineer to the company. In order to avoid the difficulties connected with the insulation of joints, when these are made in the streets Mr. Bailey makes, as far as possible, all connections of the high-pressure mains by terminal blocks on the customer's premises. Under this arrangement the insulation is only stripped at the ends which enter the terminals, and which themselves can be perfectly insulated. It is true that under this arrangement the total length of cable required is slightly increased, namely, by the length of the bight taken into each house; but this is only a small percentage of the straight run of main. Further, we have the advantage that each house is, so to speak, served by duplicate mains, namely, one on either side, and that, therefore, the house need not be cut off even if one length of main in the street should for any reason have to be disconnected. We have here, in fact, the electrical equivalent of the ring main between the engines and boilers.

2. *The Lynton Station.*—This is worked by water power from the River Lyn on a fall of 96 feet, the water being supplied to the turbine through a 30-inch pipe. Owing to the high fall the speed is

sufficient for driving the alternators, which are Mordey machines directly coupled one on either side of the turbine. Each alternator is capable of developing 37·5 kilowatts. The speed is regulated by a slide valve in the main water supply pipe worked by a hand-wheel. The mains are lead-covered Callender bitumen cables laid underground.

3. *The Keswick Station.*—This is also worked by water power obtained from the River Greta, but since the fall is only 20 feet, the alternators are driven by belt from the turbine shaft. The plant comprises two 30-kilowatt Kapp alternators, the necessary exciting machine, switchboards and instruments, and a boiler and Westinghouse engine to serve as an auxiliary source of power in case of drought or frost. The mains are insulated cables placed overhead on oil insulators, but for a certain distance they had to be taken underground, and then a Brookes' pipe line is used.

4. *The Cassel Station.*—This is an example of a central station where the motive power is electricity. There are two stations in the town in which dynamos are driven by Kapp alternators working as motors. The alternating current is supplied from

a water-power station four miles distant. Fig. 29 shows the arrangement diagrammatically. At the power station a turbine drives two Kapp alternators, each designed to give 30 ampères at 2,200

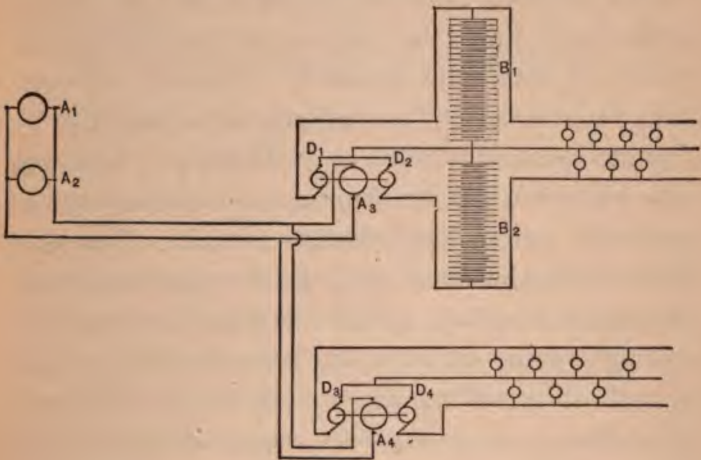



FIG. 29.

volts. The machines are coupled in parallel, and the current is taken by a concentric lead-covered cable to Cassel, where the cable splits into two branches each leading to a lighting station. At each of these lighting stations there is a 60-kilowatt alternator coupled direct to two 30-kilowatt dynamos wound for 110 volts, one dynamo on each side of the alternator. The dynamos are arranged on the three-wire system, and work on to a three-wire network common to both stations. At one of the stations there is also a battery of storage cells, from which the town is supplied during the hours of minimum demand. Towards evening, when it is necessary to supplement the batteries by dynamo power, or when it is desired to re-charge the batteries, the dynamos are switched on to the network, and receive current from it. This sets them in motion, and, working for the time being as motors, they run up the alternators to synchronizing speed. The alternating current is then switched on, and the action between the machines is reversed, the alternators acting as motors, and driving now the dynamos. The two stations supply at present current for 2,600 16-candle-power lamps burning simultaneously, or 3,500 lamps wired, but provision has been made to extend the plant, so as to eventually supply 12,000 lamps wired.

## CHAPTER X.

### PARALLEL COUPLING OF ALTERNATORS.

I have already pointed out that for economical reasons it is advisable to work the engines at a station as nearly as possible at their full load, and you will easily see that this condition can most easily be fulfilled if the alternators can be worked in parallel. For were it only possible to work each machine quite independently of the other machines, we should be obliged to keep a larger number of machines working on small loads, and as the hours of light load greatly exceed those of full load, the engines would be used under very uneconomical conditions. But if we can couple the alternators parallel, then we can put on and take off machines exactly in accordance with the demand for current, and have our engines fairly well loaded at all times. Some years ago it was believed that alternators had to be designed specially for working in parallel, and certain makers claimed this quality of their machines as something specially in their favour. If parallel running succeeded it was put down to the credit of the particular type of alternator; if it failed the design of the alternator was considered faulty. It is only recently that we have come to recognize that the real difficulty of parallel running is not in the alternator at all, but in the engine. Any alternators when driven by turbines which have an absolutely constant angular speed will run in parallel perfectly, but if you drive the machines from slow speed steam engines by means of belts or ropes, any irregularity in the angular speed of the engines is magnified by reason of the multiplication of speed, and the machine becomes alternately a generator and a motor, the transition from one state to the other being accompanied by heavy mechanical and electrical strains, which render anything like smooth working impossible. The condition of successful parallel working is, therefore, a direct-coupled engine having a very even angular speed. This is a point of great practical importance, and it is intimately connected with the general question of alternators used as motors, since when the engine fails to keep up its even angular speed the alternator steps in and compels it to do so; it acts, in fact, for a moment, as a motor, and controls the engine.

# CHAPTER XI.

## ALTERNATING CURRENT MOTORS.

When investigating the transmission of power by alternating currents, we may consider the circuit as consisting of three parts: a line having a definite resistance; an alternator working as generator at one end; and another alternator working as motor at the other end. Such a conception would be the most obvious, but it is not the best, because we are thereby compelled to investigate simultaneously the behaviour of two machines. To simplify the treatment I shall assume the following case:—Given a pair of terminals, between which by some means we maintain a constant alternating electromotive force at constant frequency, and the source from which this electromotive force is supplied shall be so abundant that we may take any amount of energy from the terminals, or put any amount of energy into them, without altering in any way either the pressure or the frequency. Such a pair of terminals would, for instance, be the omnibus bars at a central station, if from them we supply a small motor. Suppose the motor run up to synchronizing speed, and then switched on to the omnibus bars. We now want to know the relation between the mechanical power obtained, the current through the armature, and the strength of field. This apparently complex problem can be solved graphically by means of a clock diagram in a very simple manner.

It is self-evident that we can only obtain power from the motor if it runs at such a speed that the frequency of the electromotive force developed in its armature coils is exactly the same as that of the current which drives it, and that the electromotive force of the motor must be opposed to the current.

We must, therefore, at first employ some external source of power to run the motor up to the proper speed before switching on. But how are we to know when the proper speed has been reached? No tachometer or speed counter can give us this information with sufficient accuracy, especially since there may be slight variations in the frequency of the supply current. If we wish to put two alternators in parallel, we also must know exactly when their phase and frequency coincide, and for this purpose we use an instrument called a "synchronizer." It consists mainly of two small transformers (Fig. 30) the primaries of which are connected

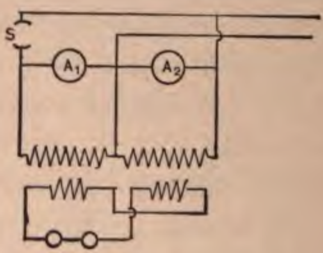



FIG. 30.

to the terminals of the two alternators which are to be coupled parallel. Two of these terminals may, of course, be permanently connected, as shown in the figure, but the other two must only be connected by the switch S when the machines are in step. The secondaries of the two transformers are connected as shown, and into this circuit are placed some incandescent lamps. By following out the connections you will easily see that if the two machines are in opposite phase, that is, in a condition when you must not couple them, there is no electromotive force on the lamps, but that when the machines are in the same phase, or, as we call it, "in step," then the lamps get the full electromotive force of the two transformers coupled in series. We thus know that when the lamps are dark the machines are out of step, and when they light up the machines are in step. But complete darkness or complete brightness can only occur when the frequencies are absolutely the same. Generally, the frequencies will be different, and the lamps will flicker. Thus, suppose one machine is running at its normal speed, and the other is being started up, at first there will be very rapid flickering in the lamps. As the speed of the second machine increases, the flickering becomes less rapid, and by degrees, namely, as the speed approaches that which is required for synchronism, there appear regular beats in the light of the lamps, which get longer and longer. You watch your opportunity, and throw the switch on in the middle of a beat when the lamps are alight. The machines are then so nearly in the right step that the first rush of current pulls them dead into step, and they remain, as it were, interlocked in that condition. The electrical coupling is, in fact, comparable to a kind of interlocking, which is as secure as if the two armature spindles were connected by spur gearing. To test the reliability of this electrical interlocking, I have run a 60 and a 10-kilowatt alternator in parallel, supplying the power from two independent sources. I have then cut off the power from the small machine. It ran on exactly as before. Next, I put a load on the small machine, and increased it to 25 horse-power; still the machine ran on. I left the load on for some hours, and then suddenly withdrew and again put on a large portion of the load, but the machine kept in step. There is, of course, for every machine a certain load at which it will be torn out of step, just the same as there is for every spur wheel a load which will strip its teeth. In the machine with which I experimented it should be possible to break down the synchronism with a load of about 30 horse-power, that is, twice the normal load, but I was not able to determine the breaking off load experimentally because the belt by which power was taken from the motor began to slip at 25 horse-power. The machines with which I experimented were of my own type, but, as I have said before, there is no particular virtue in the design, any modern alternator with a smooth armature core, and having a fair efficiency, will behave in exactly the same manner.

Having now given you some practical results of parallel running and transmission of power, I must briefly explain the theory of it. Fig. 31 represents the condition of a machine supplying current to a non-inductive resistance. OB is the electromotive force which it would at its then excitation give on open circuit, AB is the electromotive force required to overcome its self-induction with the current it actually gives, AR is the electromotive force required to overcome the armature resistance, and RO is the electromotive force available for the external circuit. If in a central station we have already a number of machines running in parallel, OR would be the electromotive force on the omnibus bars, and if we wish to switch in a new machine we would, in order to have it in the same condition as the others, excite it to such a degree that on open circuit it will give the electromotive force OB. We run the machine up to the right frequency and switch it on. For the sake of the present investigation, I will assume the new machine can be mechanically geared with one of the other machines in such a way that its electromotive force shall lag or lead in comparison with the omnibus electromotive force by any desired angle. Thus in Fig. 32 OR represents the omnibus electro-

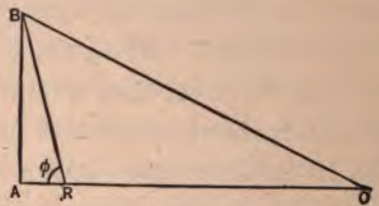



FIG. 31.

motive force, and OB the machine electromotive force, the angle between the two being ensured constant by the mechanical gearing. By drawing the parallelogram ORCB we find the resultant electromotive force in the new machine OC, and this can be regarded also as the resultant of the electromotive force of self-induction CD, and that lost in armature resistance OD. If we regard the coefficient of self-induction constant, then the angle $\varphi$, which OD makes with OC, is the same as that which in Fig. 31 RA makes with RB, and the direction of the line OD in Fig. 32 is at once defined. The

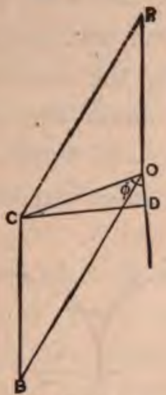



FIG. 32.

point D is found by dropping a perpendicular from C on this line. Since OD represents the electromotive force used up in resistance, and since we know the resistance, it is easy to calculate the current. We know then the direction and magnitude of the current as well as of the electromotive force, and we can find the work done by the machine. For this purpose we multiply the current with the electromotive force, and with the cosine of the angle enclosed between the two lines. The work thus found, we mark off on the line OB or its prolongation. Now let us shift our mechanical gearing and find in the same manner the current and work for a different angle between the omnibus and machine electromotive force, and repeating the construction for various phase angles, we obtain the curves on Fig. 33, which show current, and work as functions of the phase angle. The outer curve on the left repre-

FIG. 33.

sents the work given out by the machine when its phase angle lags from O to about 180°, the inner curve represents the work absorbed by the machine (when working as motor) when its phase angle lags from 180 to 360°. The curves on the right represent similarly the work given to or taken from the omnibus bars. You will notice that about half of the curves are dotted. The dotted parts refer to an unstable condition of working, and the diagram shows at a glance why it is impossible to run two alternators in series if they are independently driven, that is, not mechanically geared together, as I have assumed to be the case for the purpose of explaining how this diagram is obtained. You will also see that a moderate difference in the phase angle is sufficient to transform the machine from a strong generator into a strong motor. The difference of position in the two cases is about 90°, but you must remember that the diagram represents a two-pole machine. In reality the machines are made with many more poles, and the angle is much smaller. For instance, if there were eighteen poles the angle would only be about 10°, and this explains why it is essential for parallel working, and also for power transmission, to employ engines which will impart to the machines an almost absolutely constant angular velocity.

# CHAPTER XII.

## SELF-STARTING MOTORS.

From what I have here said you will conclude that there is no difficulty in transmitting power by a single alternating current, but that the motor is not self-starting. The system is thus encumbered by the necessity of providing a separate machine and some storage of power to set this in motion. The most convenient way is to use the exciter for this purpose, and drive it by a storage battery. When the alternator is working as a motor it drives its own exciter, and the latter may at the same time be used to charge the battery up again ready for the next start. The complication and cost of this arrangement are not very serious objections when we have to deal with large powers, but for the distribution of small parcels of power the necessity of providing a separate exciter and a storage battery, in addition to the motor proper, is a fatal objection, and various attempts have been made to design a self-starting alternate current motor. One of these, and I may at once say the most successful one, is due to Mr. Zipernowsky, whose firm (Ganz & Co., of Budapest) showed at the Frankfort Exhibition several of these machines at work. In the limited time at my disposal I cannot attempt to give you a detailed description of these, nor enter into the many refinements of construction which have been found necessary in developing the machine practically. I must content myself to give you the main principle of it. In Fig. 34 M is a laminated magnet and A an armature wound with a single coil, the ends of which are brought to a two-part commutator. It is, in fact, the well-known Siemens shuttle armature, also employed in the small Griscom motor, and the apparatus, as here shown, is nothing else than a very simple form of continuous current motor, which is self-starting from almost any position. The only position when the motor will not start is when the armature is placed so that the brush on each side touches both commutator segments at once. To start the motor from this position, it is of course necessary to slightly shift the brushes to one side or the other of the dead centre. From what I have said in Chapter I., you will easily see that this kind of


FIG. 34.

motor will also start and work with an alternating current, but its power will at first be very slight. Observe now what happens when the alternating current is switched on whilst there is no load on the motor. It will start and gather speed as all series wound motors do. If the current were continuous, the motor would very soon begin to race, but with an alternating current this cannot happen, because in trying to get up a racing pace the armature must pass through that speed which corresponds to the frequency of the supply current. At the moment when this happens, the reversal of current produced by the commutator coincides exactly with the reversal of the supply current, and the result is that the current flowing through the armature does not any more change its direction. The armature has virtually been transformed into a field magnet, excited by a continuous current, and what was at starting the field magnet has now become the armature of an ordinary alternator. The moment when the machine jumps into step can be easily noticed by the behaviour of the brushes. At starting there is violent sparking and a peculiar noise. As the machine gathers speed the sparking gets less, and suddenly there is a kind of jerk, after which both noise and sparking cease and the load may be put on. The motor, when once in step, will even stand a considerable overload.

# CHAPTER XIII.

## MULTIPHASE CURRENTS.

The motor I have just described will start itself, but it will not start with a load. The sparking is also an objection which renders the machine useless for flour mills, cotton mills, and any works where an explosion may be caused by sparks. We can therefore not regard this motor as the final solution of the problem of transmitting power by alternating currents, but must look for the solution in quite another direction. This direction has been first indicated by Professor Galileo Ferraris, of Turin, some six years ago. Quite independent of Ferraris, the same discovery was also made by Nikola Tesla, of New York; and since the practical importance of the discovery has been recognized, quite a host of original discoverers have come forward, each claiming to be the first. With these various claims we need not concern ourselves at present. I will merely describe the apparatus used by Ferraris. He employed two vertical coils AB (Fig. 35) set at right angles to each other, and a copper cylinder C suspended between them. Two alternating currents of the same frequency, but with a phase difference of 90 degrees, were sent though the two circuits, and the copper cylinder was thereby set in rotation. The explanation is as

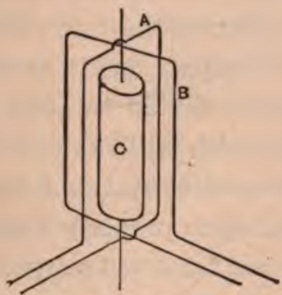



FIG. 35.

follows:—Each coil taken by itself produces an oscillating magnetic field, the lines of which are at right angles to the face of the coil. The two coils together produce a resultant field which revolves round the vertical axis of the apparatus. The surface of the copper cylinder is therefore being continuously cut by lines of force as they sweep round; currents are induced in the copper which by Lenz's law are in such directions as to resist motion; and since the cylinder is freely suspended, its endeavour to resist the motion of the field results in its being set in motion itself.

*Experiment Lantern and Model.*—In translating this laboratory experiment into practical work we must, of course, make many alterations and improvements. We must, for one thing, employ iron to get a more compact and a stronger apparatus. We must also sub-divide the two coils in order to get a machine which will run at a moderate speed, and finally we must substitute for the plain copper cylinder an armature properly wound. A machine designed on these lines will be, generally speaking, a great improvement on the original apparatus, but in one respect it will not be so good. In Fig. 35 the coils are at right angles, and the currents are at right angles. As you have seen by the model, the effect of this combination is an absolutely constant magnetic field revolving round the axis with constant speed. But if we split up the two coils into a number of sections, and wind these alternately side by side on a cylindrical core, as we wind a Gramme armature, one of our conditions, namely, that of the right angular position of the two coils, has been lost, for the coils are now very nearly in line with each other all the way round, and the result is that the field is not any more absolutely constant. I can show you this by means of the model. By setting the cranks at the wrong angle you see immediately that the vector of the resultant field is no longer the radius of a circle, but of a curve resembling an ellipse. To find the variation in the strength of the resultant field we need only draw the two current curves and add up their ordinates as I showed you in Chapter I. If you do this you will find that the maximum strength of the field is about 40 per cent. greater than its minimum value. The field is now not only a rotating one, but it also pulsates. The rotation is what we want, but the pulsation is objectionable, as, in consequence of it, useless currents are made to circulate in the armature conductors, producing heat but no power. It is the great merit of Herr von Dobrowolsky to have been the first to clearly recognize this defect in machines based on the Tesla-Ferraris motor. The evil once understood, a remedy was soon found. Dobrowolsky adopted three currents instead of two, and thus reduced the pulsation of the field at once to something like 14 per cent.; but even this was not quite satisfactory. He went, therefore, a step further and re-arranged the winding of the field in such a way as to produce the effect of six distinct currents, though still only using three wires in the line of transmission. A reference to Fig. 36 will make

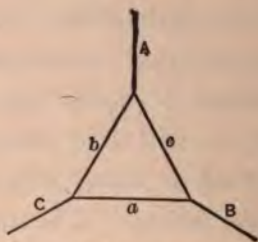



FIG. 36.

this clear. Let *abc* represent the three coils of a two-pole drum armature, such, for instance, as the armature of a Thomson-Houston machine, but instead of joining the coils to a common centre and to a three-part commutator, as is done in this type of machine, let them be joined as shown, and let the points of junction ABC be three contact rings by which the currents are received from the line. According to Kirchoff's law, the algebraical sum of the three currents ABC must at all times be ero, for if this were not the case there would be an accumulation of electricity in the machine which is obviously impossible. Any of the currents may therefore be regarded as the resultant of the other two currents. Here we have a simple three-phase winding and a rotating field, the pulsations of which are about 14 per cent. of its minimum strength. Now to reduce these pulsations, Dobrowolsky adopts the following expedient. Instead of bringing the junction between *b* and *c* direct to the contact ring A, he attaches to the junction a stouter wire and winds this round the armature in a coil placed midway between *b* and *c*. Similarly B is wound so as to split up the phase difference between *a* and *c*, and C is wound in between *a* and *b*. We have now six coils on the armature, but only half the former phase difference between neighbouring coils. Fig. 37 shows a two-pole armature so wound, and in this way the pulsation is reduced to about 4 per cent. Were the winding not split up in the manner shown, the tendency to produce fluctuations in the strength of

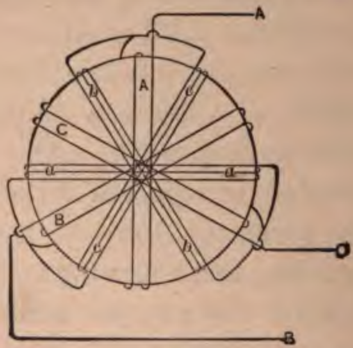


FIG. 37.

the resultant field would cause currents to circulate in the induced part of the winding, and these currents would prevent, to a certain extent, the fluctuations. But as they must necessarily circulate in coils which at the moment cannot contribute anything to the torque by reason of their position at right angles to the resultant field, these currents represent simply so much waste of power by ohmic resistance. Hence Dobrowolsky's method of splitting up the winding, although not indispensable, is a useful device for increasing the out-put obtainable from a given mass of iron and copper, and for increasing the efficiency. I have called the part of the motor represented by Fig. 37 an armature, but this was merely to point out the analogy with a Thomson-Houston armature. It would be more correct to call this part, which receives the currents from the line, the field, because its function is to produce the revolving field. The armature of the machine is a hollow cylinder of laminated iron, built up of thin plates in the usual way, and provided with a winding which is closed on itself. To understand the principle of this winding, imagine a Gramme ring, the winding of which is altered in the following way. Instead of joining the inner end of each coil with the outer end of the next coil, so as to produce a spiral winding, let the two ends of each coil be joined together. You will then have covered the Gramme core with a number of distinct coils, each closed on itself. Now put a field magnet into the inner space of the armature and revolve this magnet. The poles sweeping past the closed coils of the armature will create in them very powerful currents, and the mechanical reaction of these currents on the poles will require the application of a considerable twisting couple or torque to keep up even a moderate speed. You can test this for yourselves very easily by means of any continuous current dynamo. Excite its field separately, and short-circuit the brushes by a thick wire. If you then turn the armature by hand you will find that even exerting considerable force it will only creep round slowly, and you will thus realize how a great torque may be developed by a small angular speed of the armature in relation to the field magnet. This is an important fact, and helps us to understand two things in connection with rotary field motors. The first is that the speed of such a motor does not vary much when the load varies, since small variations of the relative speed between field and armature produce large variations in the torque; and the second is that the torque at starting is very large, the reason being that at starting the relative speed between armature and field is a maximum. It is, however, necessary to observe here that to get this large torque, resistance must be inserted in the armature circuit, for were this not done, the current in the armature coils would be so strong as to demagnetize the revolving field, thus again reducing the torque. We may now go back to Fig. 37, and see how this works out in practice. You have seen how a three-phase current passing through the winding produces a sensibly constant field, which revolves round the centre with a speed corresponding to the frequency. The armature surrounds the part shown in Fig. 37, but is omitted from the diagram. The lines of the field, in sweeping past the armature conductors, create in them very strong currents, and the mechanical reaction between these currents and the lines of the field tends to rotate the armature with great force. If the armature were movable it would thereby be set in rotation. But in the particular machine I am describing, the armature is fixed, whilst the field magnet, that is, the part shown in Fig. 37, can rotate. We have then a twisting couple between the armature and field; the armature cannot move, and therefore the field must move. Let us now see what is the effect of this movement. Say that the direction of the currents is such as to produce, when the central part of the machine is at rest, a clockwise rotation of the lines of force. The speed of rotation between the lines and the wires corresponds, of course, always to the frequency. If the wires are stationary the lines revolve in relation to any fixed object in space (for instance, the wires of the armature) with the full speed given by the frequency, say, for instance, thirty revolutions per second if the frequency is thirty and our machine is wound for two poles, as shown in Fig. 37. Each wire of the armature will therefore be cut thirty times by a north field, and thirty times by a south field in each second, and the torque produced will set the central drum rotating counter-clockwise. Say, for instance, that the central drum runs backwards with a speed of twenty revolutions per second. The relative speed between the central drum and the lines of force is, of course, still thirty revolutions per second, but of these thirty revolutions twenty revolutions are made up by the backward rotation of the drum, leaving only ten revolutions of forward speed for the lines of force in relation to any fixed point in space. The wires of the armature are now cut only ten times per second by a north field, and ten times per second by a south field. If we allow the drum to run faster still, the speed of cutting lines will be still further reduced. If, for instance, the central drum is so lightly loaded that it can acquire a speed of twenty-nine revolutions, the absolute speed of the field in relation to the armature will be reduced to one revolution, and each armature wire will be cut by a north field only once a second, and by a south field also once a second. You see, therefore, that the less the load on the motor the faster it will run, and this is precisely the same condition as obtains in an ordinary continuous current motor. At starting, when the drum is at rest, we have the greatest torque, and as the speed increases the torque diminishes. This is a very important property of the three-phase motor, since in consequence of it the machine not only becomes a self-starting motor, but one which will start with a large load. How large the starting load may be depends on the more or less skilful design of the motor. There are, as already pointed out, certain reactions of the armature on the field which tend to decrease the starting torque, but the subject is too difficult and intricate to be treated in the limited time at my disposal. I have merely given you a bare outline of the action of this class of machine, so that you may understand in a general way the principle of working.

The difference in speed of the drum and the field is technically termed the magnetic slip of the motor, and you will easily see that, to obtain a small magnetic slip, and therefore a close approach to a constant speed, we must employ an armature of small resistance. Here, again, there is a close analogy between the three-phase motor and an ordinary continuous current motor with shunt or separately excited magnets. In practice, the magnetic slip need never exceed 10 per cent., and is generally between 3 and 5 per cent. This means that the speed of the motor only varies 5 per cent. between full load and no load.

In the machine which I have described the inner revolving part is the field magnet, but you will easily understand that the design could also be reversed by making the outer ring the fixed field magnet, and the inner drum the revolving armature. This arrangement is, in fact, adopted for small motors, because in this way we avoid altogether the necessity of using rubbing contacts, but it has the disadvantage of increasing the loss from hysteresis. I have shown you that the speed with which the field sweeps through the iron of the armature is very small, namely, that corresponding to magnetic slip, whereas the speed with which the field sweeps through the iron of the field magnet is that due to the frequency, or about twenty times as great. The hysteresis loss in the armature is therefore trifling as compared with that of the field, and it is obviously of advantage to have less iron in the field than in the armature, which is done by making the inner drum the field, and the outer cylinder the armature. In small machines, where efficiency is not of paramount importance, the opposite arrangement is adopted, because of its greater simplicity and reduced cost.

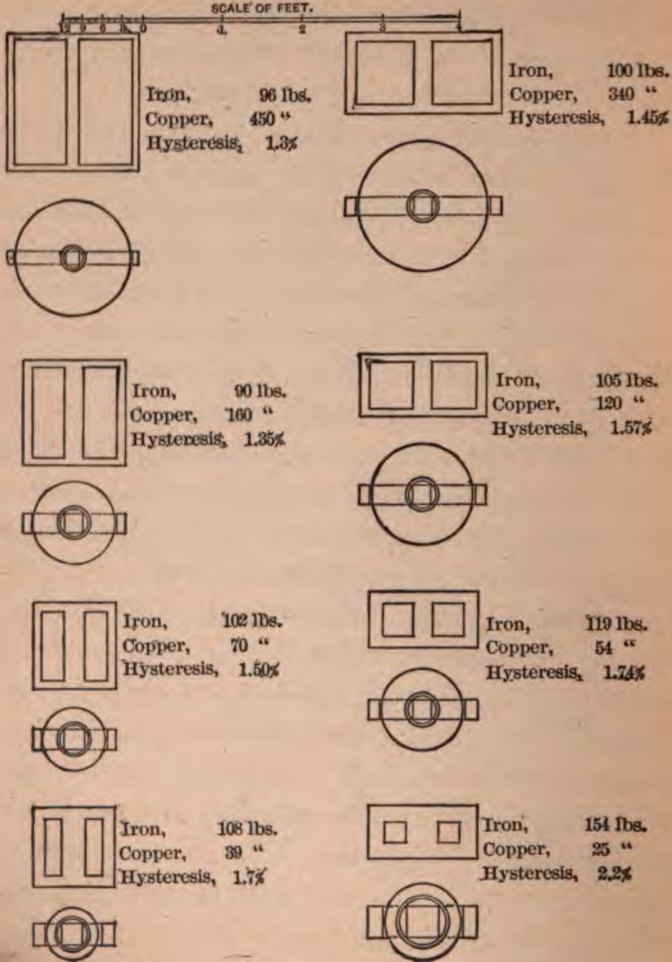
The three-phase motor has several advantages over its two rivals, the ordinary continuous current motor and the ordinary alternate current motor, whilst, in a certain measure, it combines the good qualities of both. It is better than the continuous current motor, because of its greater simplicity. There is no commutator, and there are no brushes. There can be no sparking, and the motor may therefore safely be used in coal mines and other places where a machine that is liable to sparking would be dangerous. As a matter of fact, Mr. Tesla has already constructed motors for coal-cutting machines. Its greater simplicity, and more robust construction, renders it also applicable on board ship and other places where it is exposed to rough usage. It would, for instance, be perfectly feasible to design a three-phaser which will stand being drenched with sea-water, and yet work on as if nothing had happened. Another advantage is that the distance over which power has to be transmitted can be much increased. With ordinary continuous current motors this distance is limited, because we cannot make such machines, especially if of small power, for high voltages. With a three-phaser there is no such narrow limit to the voltage, for it is always possible to work through transformers, raising the voltage at the generating station, and letting it down again at the motor station, and this can be done with very small loss. Thus, in the Lauffen transmission of power, the voltage of the generating machine was only 50 volts (measured from any of the three terminals to earth), whilst the voltage of any line wire measured in the same way was 160 times as great in some experiments, and 320 times as great in others.

Ordinary alternators offer, of course, the same facility of transmitting power at high voltage and utilizing it at low voltage, but they do not offer the same facility for distributing the power in small parcels, because each motor must be provided with some source of independent electrical energy for starting and field excitation. It is also claimed by Her von Dobrowolsky that the total weight of copper in the line is better utilized if arranged in three wires for the three-phase current than in two wires for a single-phase current, but on this point I cannot give you my own opinion, as I have not yet investigated it. One of the objections against the three-phase current is that it does not admit of a variable speed of motor, which, for many purposes, especially for traction work, is an absolute necessity. This, no doubt, is a serious drawback, but we may reasonably expect that the men who have succeeded in transmitting something like 200 horse-power over a distance of 110 miles will, in time, also succeed in solving this problem.

# SHELL TRANSFORMERS.

OUTPUT OF ALL 6 K. W.

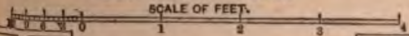
Plate I.



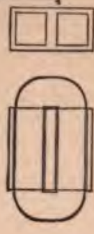
## SHELL TRANSFORMERS.

**OUTPUT OF ALL 6 K. W.**

Plate II.




Iron, 94 lbs.
Copper, 116 "
Hysteresis, 1.3%



Iron, 105 lbs.
Copper, 93 "
Hysteresis, 1.53%

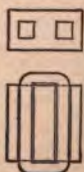
Iron, 103 lbs.
Copper, 59 "
Hysteresis, 1.55%

Iron, 114 lbs.
Copper, 48 "
Hysteresis, 1.7%

Iron, 120 lbs.
Copper, 33 "
Hysteresis, 1.77%

Iron, 135 lbs.
Copper, 27 "
Hysteresis, 1.94%

Iron, 115 lbs.
Copper, 19 "
Hysteresis, 2.24%



Iron, 173 lbs.
Copper, 15 "
Hysteresis, 2.55%

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